

Problem 1

Consider a motor governed by the differential equation,

$$\frac{dy}{dt} + \tau y(t) = u(t) \quad (1)$$

where $y(t)$ represents the speed of the motor in rpm (output), τ is a given time constant, and $u(t)$ is the input voltage applied to the motor. Let $\tau = 0.5$ seconds.

a)

Draw a block diagram of this system with closed loop control given unit negative feedback discussed in class, where y_r is the reference output or the desired speed of the motor. Label the reference output ($y_r(t)$), the actual output ($y(t)$), the input ($u(t)$), and the error ($e(t)$) on the block diagram.

b)

Use a PID (Proportional-Integral-Derivative) controller to bring the motor up to a desired CONSTANT speed, $x_r(t) = X_r$. Assuming K_d and K_i are positive, what values of K_p will ensure system stability?

c)

Now assume that after analyzing the system stability you have chosen $K_p = 35$ and $K_d = 4$. What values of K_i are necessary to ensure that the motor does not exceed the desired speed (overdamped system response)?

d)

Choose a value for K_i that satisfies the overdamped condition solved for above and explain why you made that choice. Given zero initial conditions (initial speed and acceleration of the motor are zero) and a desired motor speed of 100 rpm, solve for and plot the system response.