University of Notre Dame

Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Fall 2015
Homework 4, due September 23, 2015

1. Assume that $x_1(t)$ and $x_2(t)$ are (individually) solutions to the following ordinary, second-order differential equations. $x(t)$ is the linear combination of $x_1(t)$ and $x_2(t)$,

$$x(t) = c_1 x_1(t) + c_2 x_2(t).$$

For which of the following equation is $x(t)$ also a solution?

a. $4\ddot{x} + \dot{x} + 3x = 0.$

b. $\ddot{x} + 5\sin(t)\ \dot{x} + 2x = 0.$

c. $29\ddot{x} + 7\dot{x} + 13x = 0.$

d. $\ddot{x} + 8\dot{x} + 7x = 6t.$

What are the differences between the equations for which $x(t)$ is a solution and $x(t)$ is not a solution?

2. In the case of repeated roots of the characteristic equation,

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0,$$

prove the following two facts.

a. If the root is repeated, the value of the root is $\lambda = -\omega_n$.

b. If the root is repeated, then the two solutions

$$x_1(t) = e^{-\omega t}, \quad x_2(t) = te^{-\omega t}$$

are linearly independent.

3. Determine the solution to $7\ddot{x} - 10\dot{x} + 3x = 0$, where $x(0) = -4$ and $\dot{x}(0) = 0$.

4. Regarding variable-coefficient second-order ordinary differential equations that are easy to solve. The equation

$$t^2\ddot{x} + \alpha t\dot{x} + \beta x = 0$$

is called Euler's equation. Show that $x(t) = t^\lambda$ is a solution.

(a) Are there usually two solutions to Euler's equation? If so, are they linearly independent? If they are not linearly independent everywhere, on what intervals do they form a fundamental set of solutions?

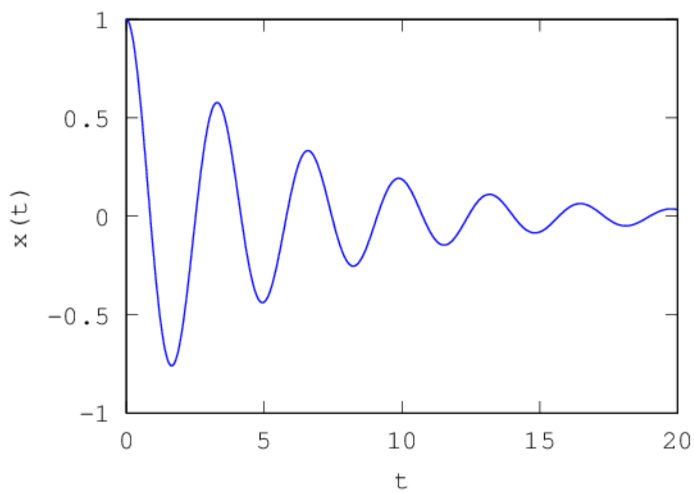
(b) Determine the general solution to

$$t^2\ddot{x} + 4t\dot{x} + 2x = 0.$$

5. The following figure illustrates the solution to

$$3\ddot{x} + \dot{x} + 11x = 0,$$

where $x(0) = 1$ and $\dot{x}(0) = 0$.



By referring to one of the forms of the solution given in Section 3.3, without solving the equation sketch what the solution will look like if

(a) The coefficient of $\ddot{x}$ is increased

(b) The coefficient of $\ddot{x}$ is decreased

(c) The coefficient of $\dot{x}$ is increased a little

(d) The coefficient of $\dot{x}$ is increased a lot

(e) The coefficient of $\dot{x}$ is decreased

(f) The coefficient of x is increased

(g) The coefficient of x is decreased

Verify your predictions by solving the equation and plotting the solution. Using a computer package is acceptable. Insight from problems of this type is very useful in designing feedback controllers in AME 30315.