University of Notre Dame

Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Fall 2015
Homework 9, due November 11th, 2015

Problem.1

Compute the Fourier series for each of the following functions.

(a) $f(x) = f(x + 10)$ and $f(x) = \begin{cases} -1, & 0 < x \leq 5, \\ 9, & 5 < x \leq 10. \end{cases}$

(b) $f(x) = f(x + 2)$ and $f(x) = \begin{cases} x^2, & 0 < x \leq 1, \\ (x-1)^2, & 1 < x \leq 2. \end{cases}$

Problem.2

Determine the solution to the one-dimensional wave equation $\dfrac{\partial^2 u}{\partial t^2} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}$ with $u(0, t) = u(L, t) = 0$ and with the specified parameter values and initial conditions in each of the following cases:

(a) $\alpha = 1$, $L = 10$, $\dfrac{\partial u}{\partial t}(x, 0) = 0$, and $u(x, 0) = \begin{cases} \dfrac{x}{10}, & 0 < x \leq 5, \\[2mm] \dfrac{10 - x}{10}, & 5 < x \leq 10. \end{cases}$

(b) $\alpha = 2$, $L = 10$, $u(x, 0) = 0$, and $\dfrac{\partial u}{\partial t}(x, 0) = \begin{cases} 0, & 0 < x \leq 1, \\ 1, & 1 < x \leq 2, \\ 0, & 2 < x \leq 10, \end{cases}$