University of Notre Dame

Aerospace and Mechanical Engineering

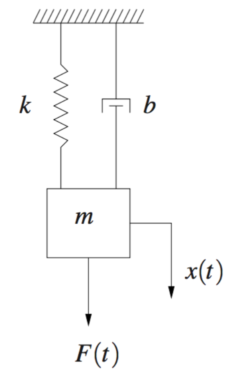
AME 30314: Differential Equations, Vibrations and Controls I
Fall 2015
Homework 10, due November 20$^{\text{th}}$, 2015

Problem.1

Consider the mass-spring-damper system illustrated in the figure shown.
$m = 1$, $b = 0$ and $k = 4\pi^2$.
Let

$$f(t) = \begin{cases} 1, & 0 \leq t < 1, \\ 0, & 1 \leq t < 2. \end{cases}$$

and $f(t+2) = f(t)$. Also, let $x(0) = \dot{x}(0) = 0$.



(1) Determine the solution by considering each interval of time $t_n \leq t \leq t_n + 1$, where $t_n = 0, 1, 2, ...$ separately. Find the solution for $0 \leq t < 1$. Then using the value of that solution at $t = 1$ for the initial conditions for the next interval, determine the solution for $1 \leq t < 2$.

(2) Write a computer program to determine an approximate numerical solution for this system and compare the answer to the answer from part (1).

(3) Expand $f(t)$ in a Fourier series and use the method of undetermined coefficients to find the solution. Hint: The solution will be a series. Plot the solution for including various numbers of terms in the series solution and compare it to the numerical solution in part (2).

Problem.2
Determine the solution to the one-dimensional heat condition equation with homogeneous boundary conditions and with the specified parameter values and initial condition:

$$\alpha = 1, \quad L = 10, \text{ and } u(x,0) = \begin{cases} \dfrac{x}{5}, & 0 < x \leq 5, \\[2ex] \dfrac{10-x}{5}, & 5 < x \leq 10. \end{cases}$$

Plot a partial sum of the solution including enough terms to that it is accurate for various times.

Problem.3
Show that the the eigenvalue for the one-dimensional heat conduction equation with homogeneous boundary conditions must be positive.

Problem.4
Determine the solution to Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

where

$$u(0,y) = 0, \quad u(L_x,y) = 0, \quad u(x,0) = f(x), \quad \text{and} \quad u(x,L_y) = 0.$$

Problem.5
Show that if the thermal conductivity is not uniform throughout the body, then the heat conduction equation in Cartesian coordinates in three dimensions is

$$\frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial u}{\partial z}\right) = \rho c_p \frac{\partial u}{\partial t}$$

Problem.6
Determine the solution to

$$\alpha^2\frac{\partial^2 u}{\partial x^2} - \alpha^2 u = \alpha^2\frac{\partial^2 u}{\partial t^2},$$

where

$$u(0,t) = u(L,t) = 0,$$

and

$$u(x,0) = f(x), \frac{\partial u}{\partial t}(x,0) = 0.$$