University of Notre Dame

Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I

Fall 2015

Homework 11

Problem.1

For the following initial value problems, write a computer program that uses Euler's method to determine an approximate numerical solution.

$$\dot{x} + 6x = \sin t, x(0) = 0.$$

Be sure to reduce the time step until the solution seems to converge. If it is possible to determine the exact solution using methods from the textbook, do so and compare the exact and approximate solutions, and in this case, if the time step is reduced, does the error decrease as expected?

Problem.2

Choose your favorite ordinary differential equation for which you can find the exact solution. If you are working with others, you must choose a different equation than they do. The only other condition is that it must be of the form $\dot{x}(t) = f(x, t)$ and f must depend on BOTH x and t. It can be a single first order equation, or a higher-order equation converted to a system of first order equations. Write a program (or separate programs) that use

(a) Euler's method.

(b) Second order R-K.

(c) Fourth order R-K.

Plot the error versus different time steps. Indicate whether the error is decreasing in a manner consistent with the order of the method.

Problem.3

Consider

$$\ddot{x} + \dot{x} + x = \sin t$$

and

$$\ddot{x} + \dot{x} + x = \sin 100t$$

where $x(0) = 0$ and $\dot{x}(0) = 0$. Write computer programs that use Euler's method to compute approximate numerical solutions for each one and run it using the following time steps, $\Delta t = 0.1$, $\Delta t = 0.01$ and $\Delta t = 0.001$. Continue to reduce the time step until the numerical solution is a good approximation for the exact solution. Explain any unusual aspects of this problem. Would using a higher-order method eliminate these unusual aspects?

Problem.4

Consider the differential equation, which does not look all that bad:

$$\frac{dx}{dt} = 40x(1 - x),$$

where

$$x(-1) = \frac{1}{1 + e^{40}}.$$

Note that the initial condition is at $t = -1$. Use MATLAB and `ode45()` to solve this. Compare it to the exact answer, which is

$$x(t) = \frac{1}{1 + e^{-40t}}$$

by plotting the two on the same graph. Verify the given exact answer really is the answer by substituting it into the differential equation. On a different graph, plot the error. Does MATLAB give a good solution?

Write a program using 4th order Runge-Kutta to determine the solution. Plot the error versus different sized time steps.

Main point: can you always trust MATLAB to give a good answer? Look at the original equation. Does it look suspicious in any way that would lead you to believe that it is problematic? (My answer is NO, unless you ponder it for a long time).