University of Notre Dame
Aerospace and Mechanical Engineering

AME 302: Differential Equations, Vibrations and Controls II
Exam 3

B. Goodwine
April 26, 2004

NAME: ______________________________

- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, your class notes, your own homework sets and any documents provided on the course homepage such as homework solutions, *etc.*
- You may use a calculator **only** to compute values for trigonometric functions, inverse trigonometric functions and simple arithmetic such as multiplication. You may not use a calculator for any other purpose.
- There are four questions. Problems 1 and 3 are worth 30 points and problems 2 and 4 are worth 20 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page or on the blank pages. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- Do not start or turn the page until instructed to do so.

I would rather discover a single fact, even a small one, than debate the great issues at length without discovering anything at all.

Galileo Galilei

1. Plot the root locus plot for

$$G(s) = \frac{s+1}{(s+3)(s^2+2s+2)}$$

which shows how the poles of feedback system in Figure 1 vary as the parameter K varies from 0 to $+\infty$. For full credit, be sure to show your computations for

(a) asymptote angles;

(b) the intersection point on the real axis for the asymptotes;

(c) departure angles and/or arrival angles for complex conjugate poles and zeros.

Hint: in this problem, there are no points where the root locus breaks into or breaks out from the locus in the real axis, so you may skip any computations related thereto. (30 points)

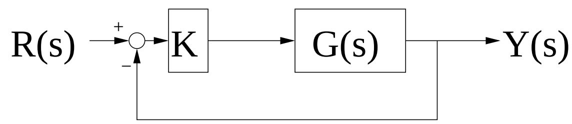

Figure 1. Feedback system for Problem 1.

Intentionally left blank.

2. The root locus plot for

$$G(s) = \frac{1}{s^2 + 2s + 2}$$

is illustrated in Figure 2. For this $G(s)$ and the feedback system illustrated in Figure 1, determine

(a) an approximate value of K that will yield a maximum percentage overshoot of $M_p = 23\%$;

(b) an approximate value of K that will yield a rise time of 0.9 seconds; and,

(c) an approximate range of values of K, if possible, that will satisfy both criteria, *i.e.*, a maximum percentage overshoot less than 23% and rise time of less than 0.9 seconds.

Where possible, it is appropriate to determine approximate values of pole locations by simply referring to Figure 2. (20 points)

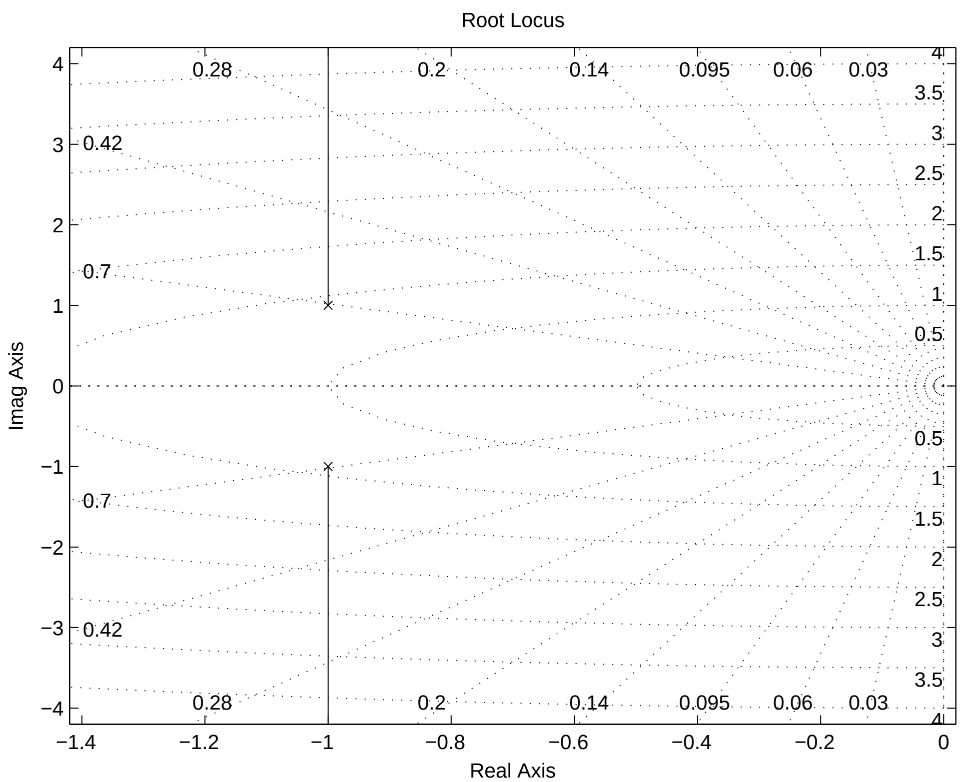

Figure 2. Root locus plot for Problem 2.

Intentionally left blank.

3. On the following graphs, plot the Bode plot for the transfer function

$$G(s) = \frac{10000(s+10)}{(s+1)(s^2+100s+10000)}.$$

On Figure 3, plot the contribution of each of the individual terms in the transfer function. Be sure to label which term corresponds to which curve. On Figure 4 plot the overall Bode plot (the sum of all the terms in Figure 3). On on the magnitude plots, be sure to label the slope of the curves unless the slope is obviously zero. (30 points)

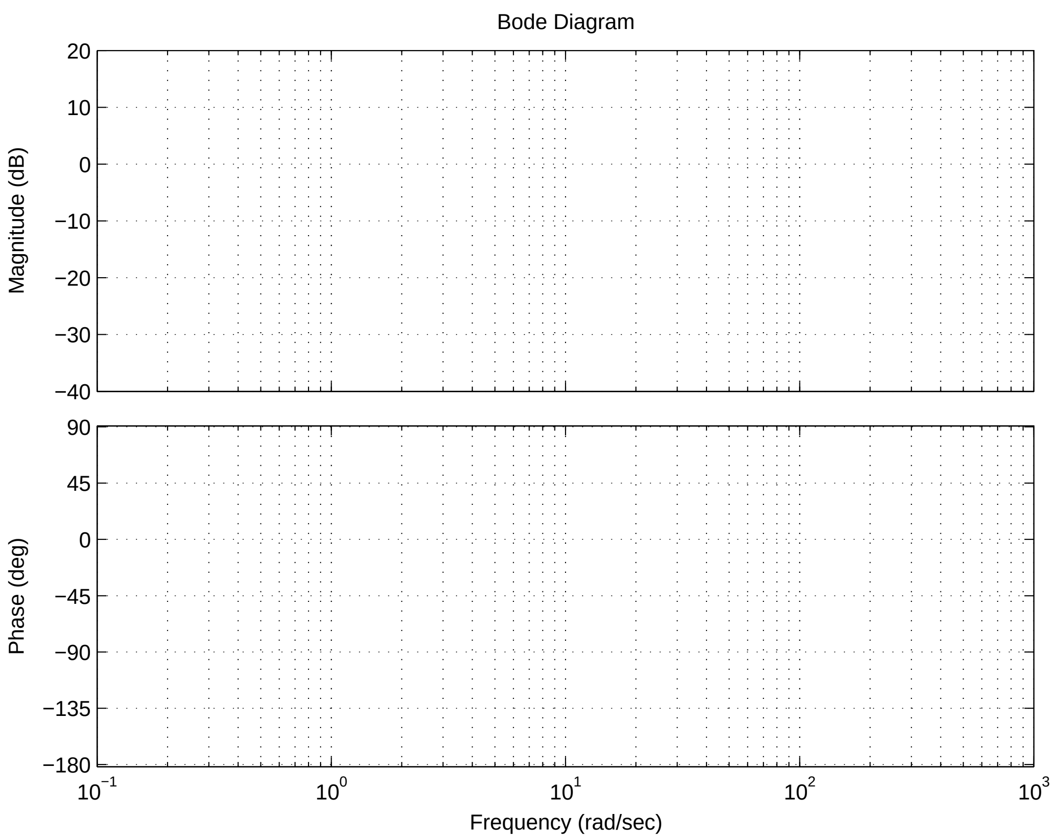

Figure 3. Bode plot for Problem 3.

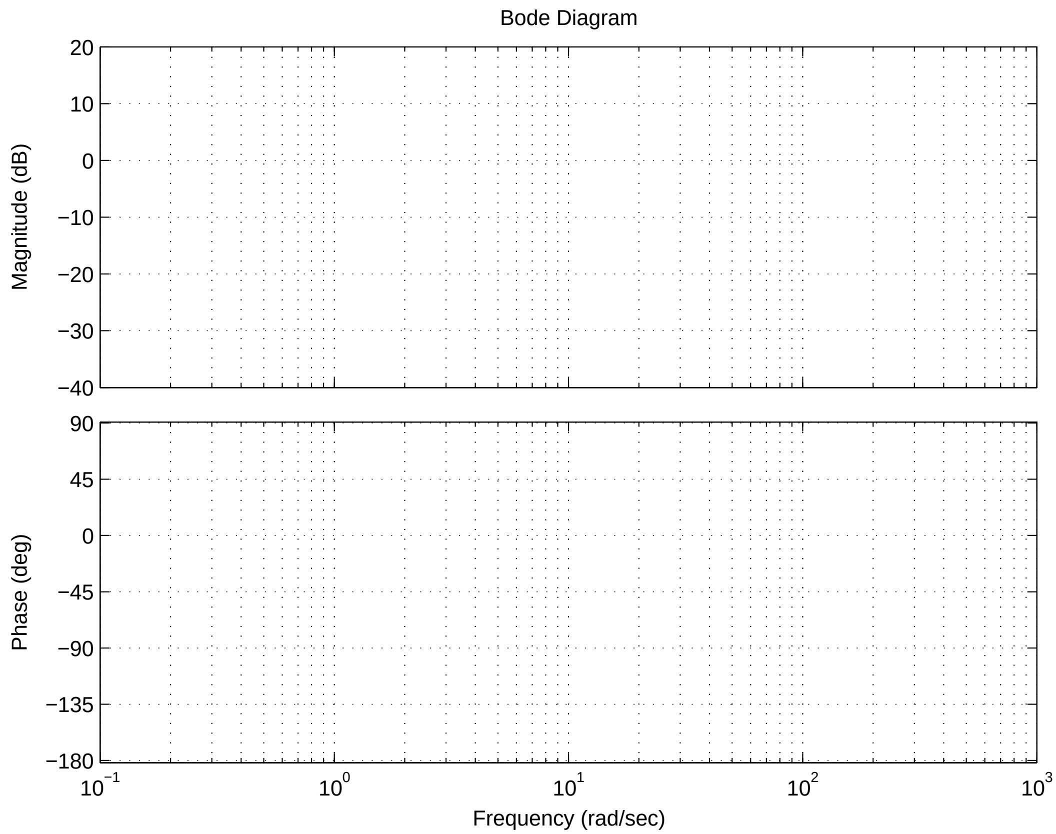

Figure 4. Bode plot for Problem 3.

4. Consider the Bode diagram for

$$G(s) = \frac{100}{(s+10)(s^2+2s+2)}$$

illustrated in Figure 5.

(a) Will the system be stable under unity feedback?

(b) By referring to Figure 5, determine approximate values for the gain and phase margins.

(20 points)

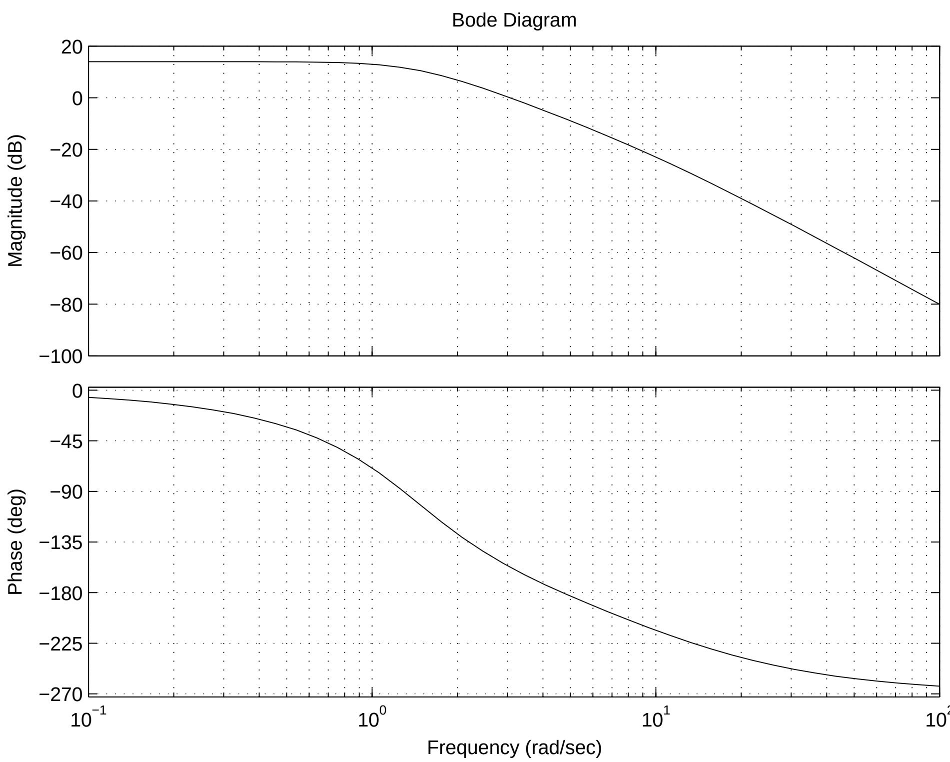

Figure 5. Bode plot for Problem 4.