University of Notre Dame
Aerospace and Mechanical Engineering

AME 302: Differential Equations, Vibrations and Controls II
Exam 1

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NAME: ______________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course texts, any other text book, your class notes, homework solutions and your own homework sets.
- You may **not** use a calculator.
- There are four problems, each of which are worth 25 points.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

Everything on earth is subject to change. Prosperity is followed by decline: this is the eternal law on earth. Evil can indeed be held in check but not permanently abolished. It always returns. This conviction might induce melancholy, but it should not; it ought to only to keep us from falling into illusion when good fortune comes to us. — *The I Ching or Book of Changes*

1. Find the solution to

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix},$$

where

$$\xi(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

2. Determine four linearly independent solutions to $\dot{\xi} = A\xi$ where

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

3. Consider

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ 12 & -3 \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 1 \\ e^t \end{bmatrix}. \tag{1}$$

(a) Determine the general solution to equation 1 using the diagonalization method.

(15 points)

(b) If you were to use the method of undetermined coefficients to solve equation 1, what would you assume for the form of the particular solution? Do not actually compute the coefficients – just give what you would start with for the particular solution. (5 points)

(c) What is the fundamental matrix solution, $\Xi(t)$ for the system in equation 1? (5 points)

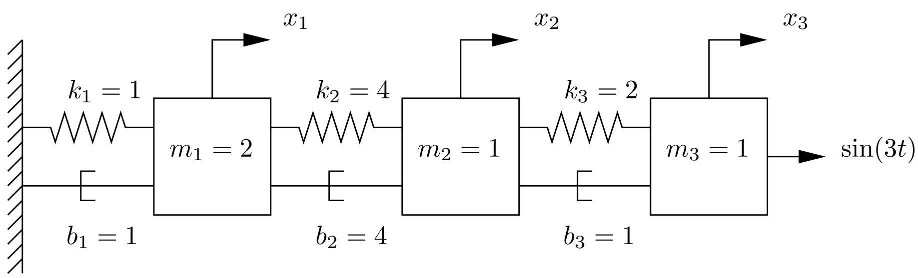

Figure 1. System for problem 4.

4. Consider the system illustrated in figure 1, which consists of three mass–spring–damper systems with a force of $\sin(3t)$ exerted on the end mass.

   (a) Determine the equations of motion for the system.

(b) Convert the equations of motion to a system of first order ordinary differential equations.

(c) Using either the C or FORTRAN code that follows, fill in the blanks so that the program will implement Euler's method to determine an approximate numerical solution to the system illustrated in figure 1 assuming that $x_1(0) = 0, \dot{x}_1(0) = 0, x_2(0) = 0, \dot{x}_2(0) = 0, x_3(0) = 0$ and $\dot{x}_3(0) = 0$.

```
#include<stdio.h>
#include<math.h>

void main() {

  double t,dt,tfinal,x[6],m[3],k[3],b[3];
  FILE *fp;

  dt = 0.001;
  tfinal = 100.0;
  m[0] = 2;
  m[1] = 1;
  m[2] = 1;
  k[0] = 1;
  k[1] = 4;
  k[2] = 2;
  b[0] = 1;
  b[1] = 4
  b[2] = 1;

  x[0] = 0.0;
  x[1] = 0.0;
  x[2] = 0.0;
  x[3] = 0.0;
  x[4] = 0.0;
  x[5] = 0.0;

  fp = fopen("data.d","w");

  for(t=0.0;t<=tfinal;t+=dt) {
    fprintf(fp,"%f\t%f\t%f\n",t,x[0],x[2]);
    x[0] += _______________________________________;
    x[1] += _______________________________________;
    x[2] += _______________________________________;
    x[3] += _______________________________________;
    x[4] += _______________________________________;
    x[5] += _______________________________________;
  }

  fclose(fp);

}
```

```
      program exam1

      double precision x(6),t,dt,tfinal,m(3),k(3),b(3)

      dt = 0.001
      tf = 50.0

      m(1) = 2
      m(2) = 1
      m(3) = 1
      k(1) = 1
      k(2) = 4
      k(3) = 2
      b(1) = 1
      b(2) = 4
      b(3) = 1

      x(1) = 0.0
      x(2) = 0.0
      x(3) = 0.0
      x(4) = 0.0
      x(5) = 0.0
      x(6) = 0.0

      open(u,FILE='data.d',STATUS='OLD')

      do 10 t=0.0,tfinal,dt
         write(u,*) t,x(1),x(2),x(3),x(4),x(5),x(6)
         x(1) = x(1) + ________________________________
         x(2) = x(2) + ________________________________
         x(3) = x(3) + ________________________________
         x(4) = x(4) + ________________________________
         x(5) = x(5) + ________________________________
         x(6) = x(6) + ________________________________

10    continue

      close(u)
      stop
      end
```