University of Notre Dame
Aerospace and Mechanical Engineering

AME 302: Differential Equations, Vibrations and Controls II
Exam 2

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NAME: ______________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course texts, any other text book, your class notes, homework solutions and your own homework sets.
- You may **not** use a calculator.
- There are three problems. Problem 1 is worth 50 points, problem 2 is worth 30 points, problem 3a is worth 20 points and problem 3b is optional and is worth 10 points extra credit.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

It is not the critic who counts: not the man who points out how the strong man stumbles or where the doer of deeds could have done better. The credit belongs to the man who is actually in the arena, whose face is marred by dust and sweat and blood, who strives valiantly, who errs and comes up short again and again, because there is no effort without error or shortcoming, but who knows the great enthusiasms, the great devotions, who spends himself for a worthy cause; who, at the best, knows, in the end, the triumph of high achievement, and who, at the worst, if he fails, at least he fails while daring greatly, so that his place shall never be with those cold and timid souls who knew neither victory nor defeat.

— Theodore Roosevelt
"Citizenship in a Republic"
April 23, 1910, Paris.

1. Solve

$$\ddot{x} + 4x = f(t) \qquad x(0) = 0, \quad \dot{x}(0) = 0$$

where

$$f(t) = \begin{cases} -4 & 0 \le t < \pi \\ -4\,(t-\pi) & t \ge \pi \end{cases}$$

using Laplace transforms.

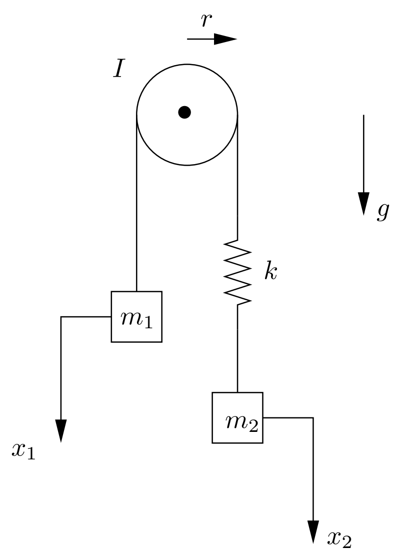

Figure 1. System for Problem 2.

2. Consider the system illustrated in Figure 1. It consists of two masses connected together with a belt over a pulley. On the right part of the system, the belt is elastic, and can be considered a linear spring with spring constant k. The masses have mass m_1 and m_2 respectively, and the pulley has a moment of inertia, I and radius r.

   You may assume that the positions of the masses, x_1 and x_2 respectively are measured from an equilibrium position and that the belt does not slip on the pulley.

   Use Lagrange's equations to determine the equations of motion for the system.

3. Consider the system illustrated in Figure 2.

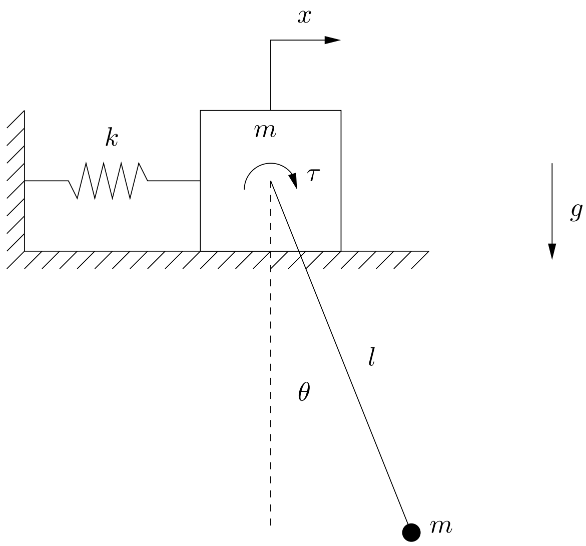

Figure 2. System for Problem 3.

In homework 7 you determined that the equations of motion for the system illustrated in Figure 2 were

$$\begin{aligned} 2m\ddot{x} + ml\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right) + kx &= 0 \\ ml^2\ddot{\theta} + ml\ddot{x}\cos\theta + mgl\sin\theta &= \tau. \end{aligned}$$

If θ and $\dot{\theta}$ are small, then an approximation to the equations is

$$2m\ddot{x} + ml\ddot{\theta} + kx = 0 \quad (1)$$

$$ml^2\ddot{\theta} + ml\ddot{x} + mgl\theta = \tau. \quad (2)$$

(a) Using equations 1 and 2 determine the transfer function from the input torque, $\tau(t)$ to the position of the mass, $x(t)$.

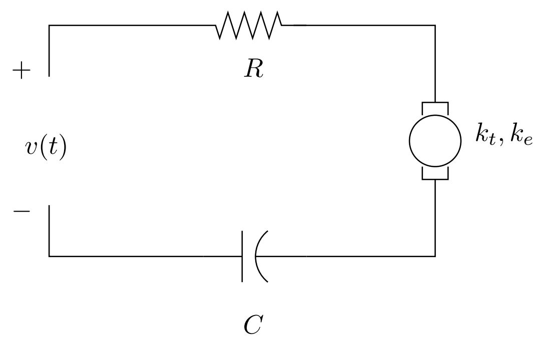

Figure 3. Motor circuit for Problem 3.

(b) If the torque, τ is applied by a d.c. motor driven by the circuit illustrated in Figure 3, determine the transfer function from the applied voltage. $v(t)$ to the position of the mass, $x(t)$.