University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
Second Exam

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NAME: ______________________________

- Do not start or turn the page until instructed to do so.
- You have 120 minutes to complete this exam.
- You may consult the course text, your class notes, homeworks and homework solutions.
- You may **not** use a calculator or other electronic device.
- There are four problems, each of which is worth 25 points.
- If any of your answers involve complex numbers, you must convert the answer to a form that is purely real.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

I was ahead in the slalom. But in the second run, everyone fell on a dangerous spot. I was beaten by a woman who got up faster than I did. I learned that people fall down, winners get up, and gold medal winners just get up faster.

— Bonnie St. John

1. Solve

$$\begin{aligned} \ddot{x} + 2\dot{x} + 4x &= \cos\sqrt{3}t \\ x(0) &= 0 \\ \dot{x}(0) &= 1 \end{aligned}$$

using Laplace transforms.

2. Determine the solution to

$$\begin{aligned}
\ddot{x} + x &= \begin{cases} 0, & 0 \le t < \pi \\ 1, & \pi \le t < 2\pi \\ 2, & 2\pi \le t \end{cases} \\
x(0) &= 1 \\
\dot{x}(0) &= 0.
\end{aligned}$$

Also sketch the solution versus time.

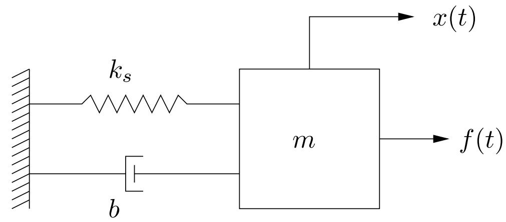

Figure 1. System for Problem 3.

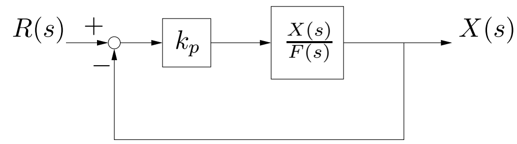

Figure 2. Block diagram for Problem 3.

3. Consider the system illustrated in Figure 1.

   (a) Find the transfer function from the applied force, $f(t)$ to the position of the mass, $x(t)$.

   (b) Consider the block diagram illustrated in Figure 2. Using your answer from part 3a, determine the transfer function from the input, $r(t)$ to the output, $x(t)$.

   (c) Let $m = k = b = 1$, $R(s) = \frac{1}{s}$ and consider the two cases where $k = 1$ and $k = 2$.

      i. Which k value will have a faster rise time?

      ii. Which k value will have a greater percentage overshoot.

      iii. Which k value will have a faster settling time?

   In each case, explain your answer.

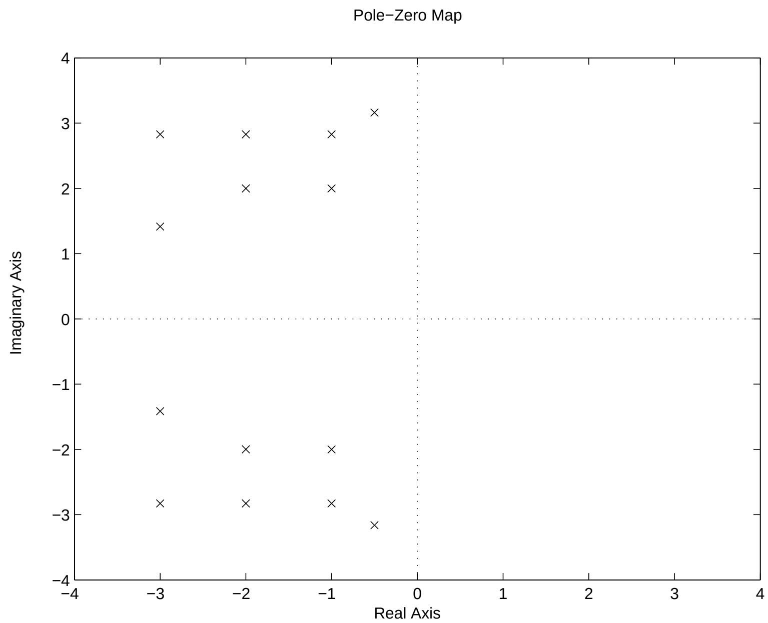

Figure 3. Pole locations for Problem 4.

4. Consider the pole locations illustrated in Figures 3 and 4 (the figures are the same). Considering the complex conjugate pairs of poles individually in response to a unit step input:

   (a) in Figure 3, circle the pole with the shortest rise time and put a square around the pole with the longest rise time;

   (b) in Figure 3, put a triangle with a point "up" around the pole with the shortest settling time and put a triangle with a point "down" around the pole with the longest settling time;

   (c) in Figure 3, put a star on top of the pole with the smallest percentage overshoot and a pentagon around the pole with the largest percentage overshoot;

   (d) in Figure 4, circle all the poles that have a settling time less than 1.25 seconds. *Hint:* $\ln(0.05) \approx -3$; and,

   (e) in Figure 4, put a square around all the poles that have a rise time less than .2 seconds.

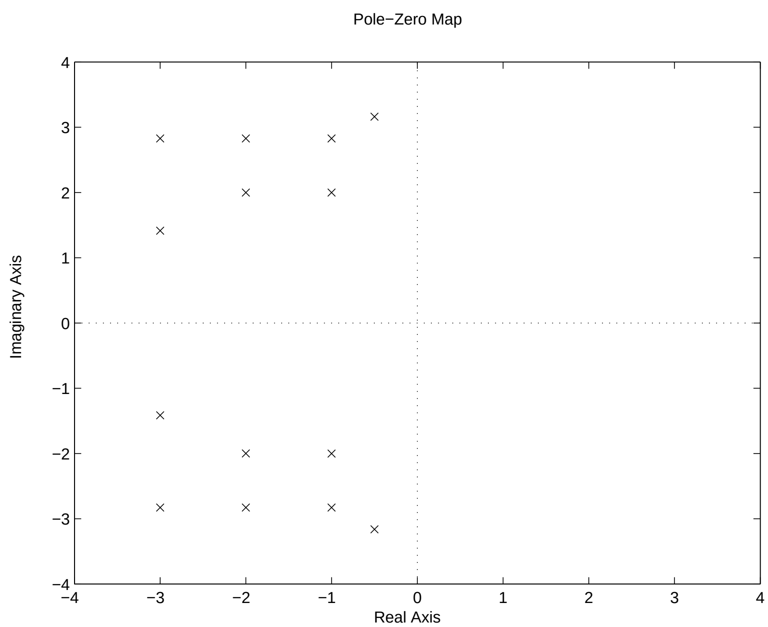

Figure 4. Pole locations for Problem 4.