University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
First Exam

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ID Number:______________________________

NAME:______________________________

- Do not start or turn the page until instructed to do so.
- You have 120 minutes to complete this exam.
- This is an open book exam. You may consult the course text and anything you have written in it, but nothing else.
- You may **not** use a calculator or other electronic device.
- There are four problems, each worth 25 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

Top results are reached only through pain. But eventually you like this pain. You'll find the more difficulties you have on the way, the more you will enjoy your success.

—Juha "The Cruel" Väätäinen

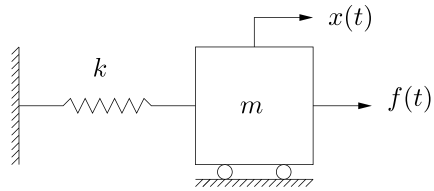

Figure 1. System for Problem 1.

1. Consider the mass-spring system illustrated in Figure 1 and let $m = k = 1$.

   - Determine the equations of motion and convert them to the form

   $$\dot{\xi} = A\xi + g(t).$$

   - Let the force, $f(t)$ be of the form

   $$f(t) = k_1 x + k_2 \dot{x}.$$

     Determine the values for k_1 and k_2 such that the eigenvalues for the resulting system are $\lambda_1 = -2$ and $\lambda_2 = -4$.

   - Solve the resulting system if $x(0) = 1$ and $\dot{x}(0) = 0$. On the same plot sketch x and $\dot{x}$ *versus* t.

2. Determine the solution to $\dot{\xi} = A\xi$ where

$$A = \begin{bmatrix} -3 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

and

$$\xi(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

3. Consider

$$\frac{d}{dt}\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\end{bmatrix} = \begin{bmatrix}-3 & 0 & 0\\ 0 & 0 & 3\\ 0 & -3 & 0\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \cos 3t\end{bmatrix}.$$

- Determine the homogeneous solution.
- If you were to use undetermined coefficients, what is the appropriate form to assume for the particular solution? Do not solve it (unless you want to do so as a check on your answer, but you will not receive any credit for solving it).

4. Use Laplace transforms to determine the solution to

$$\ddot{x} + 3\dot{x} + 2x = e^{-2t} \tag{1}$$

$$x(0) = 1 \tag{2}$$

$$\dot{x}(0) = -1. \tag{3}$$