University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
Second Exam

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March 26, 2010

ID Number:________________________________

NAME:________________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and anything you have written in it, but nothing else.
- You may **not** use a calculator or other electronic device.
- There are three problems. The first two problems are worth 30 points and the third problem is worth 40 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

Nothing splendid has ever been achieved except by those who dared believe that something inside them was superior to circumstance. –Bruce Barton

1. Determine *and sketch* the solution to

$$\dot{x} + 2x = \begin{cases} 0, & 0 \le t < 1, \\ 2, & 1 \le t < 2, \\ 0, & t \ge 2 \end{cases}$$

for $t \ge 0$.

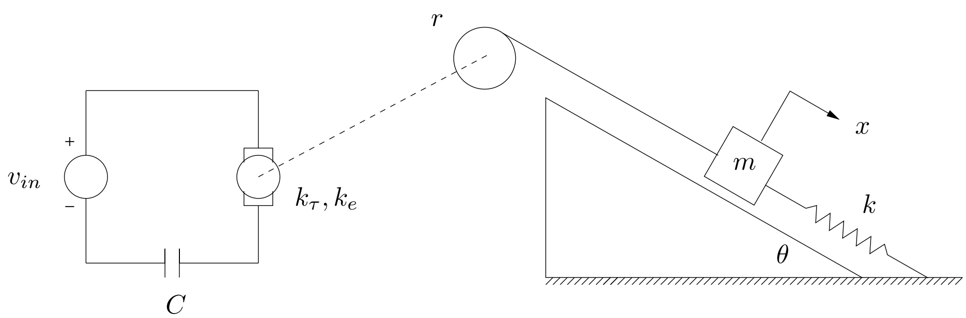

Figure 1. System for Problem 2.

2. Consider the gizmo illustrated in Figure 1, where a dc motor drives a pulley that is attached to a mass with a belt. Assume

   - there is no gravity, or it can be ignored;
   - the pulley, belt and motor shaft are light with negligible inertia;
   - the spring is unstretched when $x = 0$; and,
   - the belt from the pulley to the mass can exert both a positive and negative force on the mass.

   Find the transfer function from the input voltage to the position of the mass.

3. Consider

$$G(s) = \frac{s+2}{(s-1)(s+1)(s+3)}.$$

(a) Sketch the root locus plot for this transfer function, *i.e.*, all the solutions for $1+kG(s) = 0$ for positive k.

You must include the computations supporting each feature in the plot. Plots with no computations will be treated as lame attempts at partial credit, and for whatever reason I'm in no mood for that, so such plots will receive *negative* credit.

(b) Consider the block diagram illustrated in Figure 9.97 on page 340 in the course text with

$$C(s) = \frac{s+2}{s+3}, \quad G(s) = \frac{1}{(s-1)(s+1)}.$$

Using your root locus plot, for what range of values of k will the system have a step response that is stable, *i.e.*, decaying?