University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
Third Exam

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April 28, 2010

ID Number:______________________________

NAME:______________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and anything you have written in it, but nothing else.
- You may **not** use a calculator or other electronic device.
- There are four problems. The first three problems are worth 30 points and the fourth problem is worth 10 points. While not absolutely necessary, the problems sort of build on each other, so initially you should probably try to do them in order.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

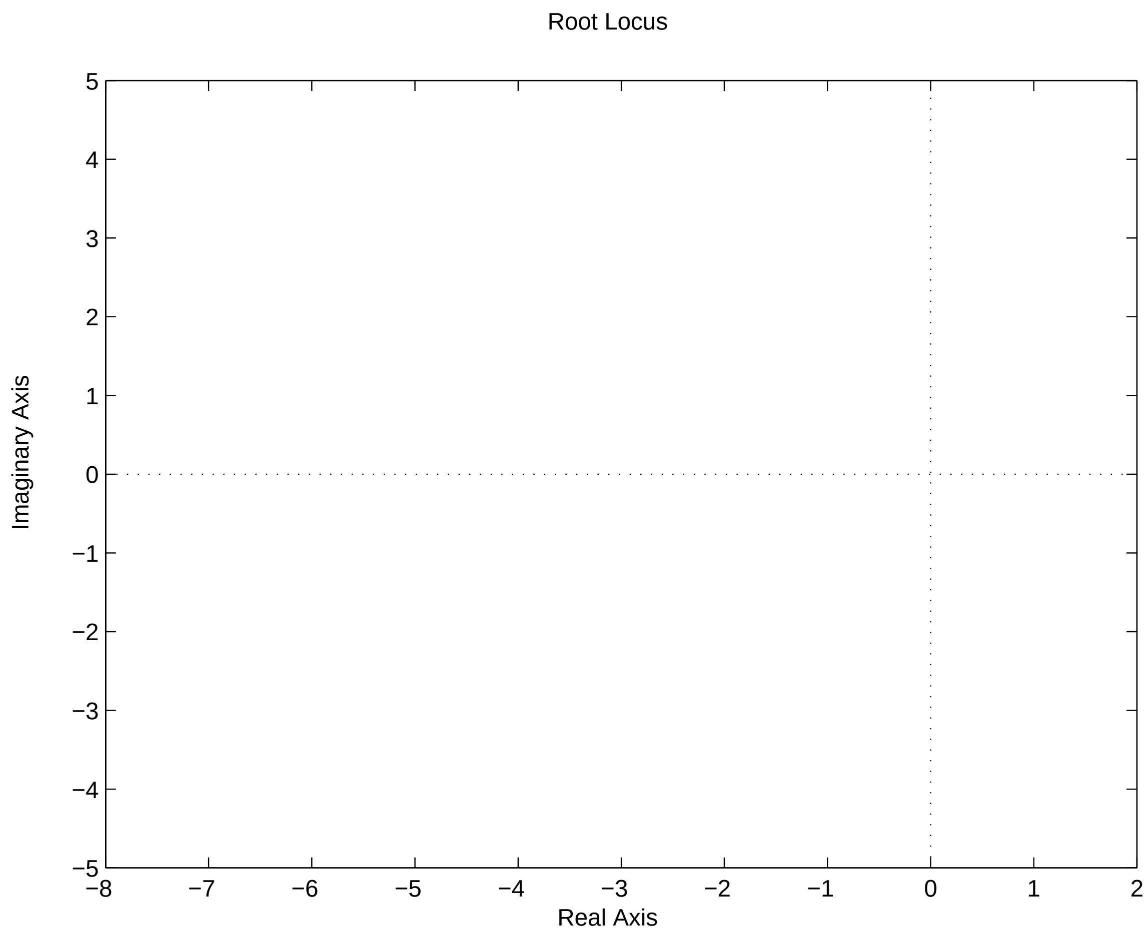
Are you sure you want to test your limits? 'Tis much more popular to limit your tests.

–Lazarus Lake

1. Consider

$$G(s) = \frac{4}{(s+2)\left(s^2+2s+2\right)}.$$

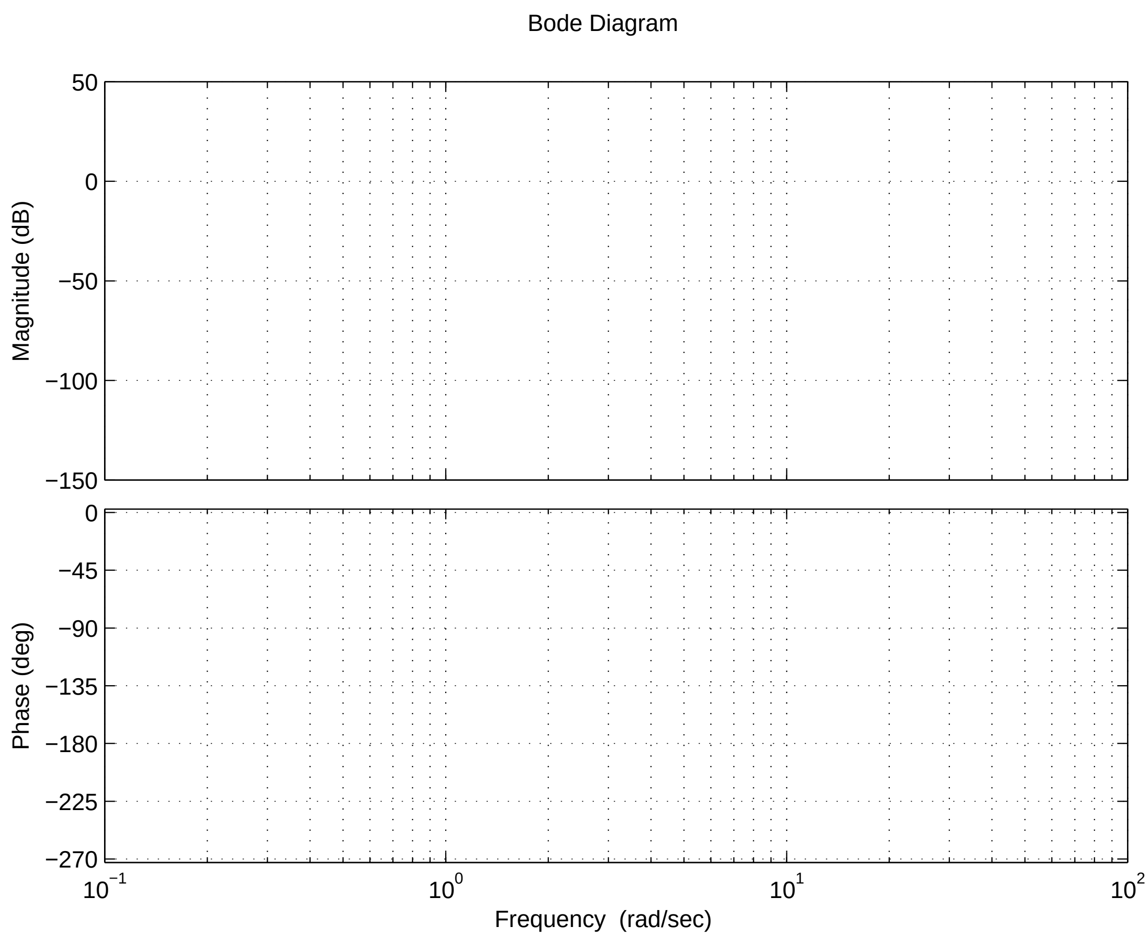
- On the axes on the following page, neatly sketch the root locus plot for $G(s)$. Be sure to include computations, if applicable, for each step in the root locus plotting process.
- From your root locus plot, determine the range of values for k for which the system placed in the feedback loop illustrated in Figure 9.44 of the course text is stable.
  - You do not need to do any elaborate trigonometry by hand – just make a neat plot and visually estimate some of the distances.
  - Indicate on the plot what the distances you used and what you used for the values.
  - You do not need to compute exactly where the root locus crosses the imaginary axis.

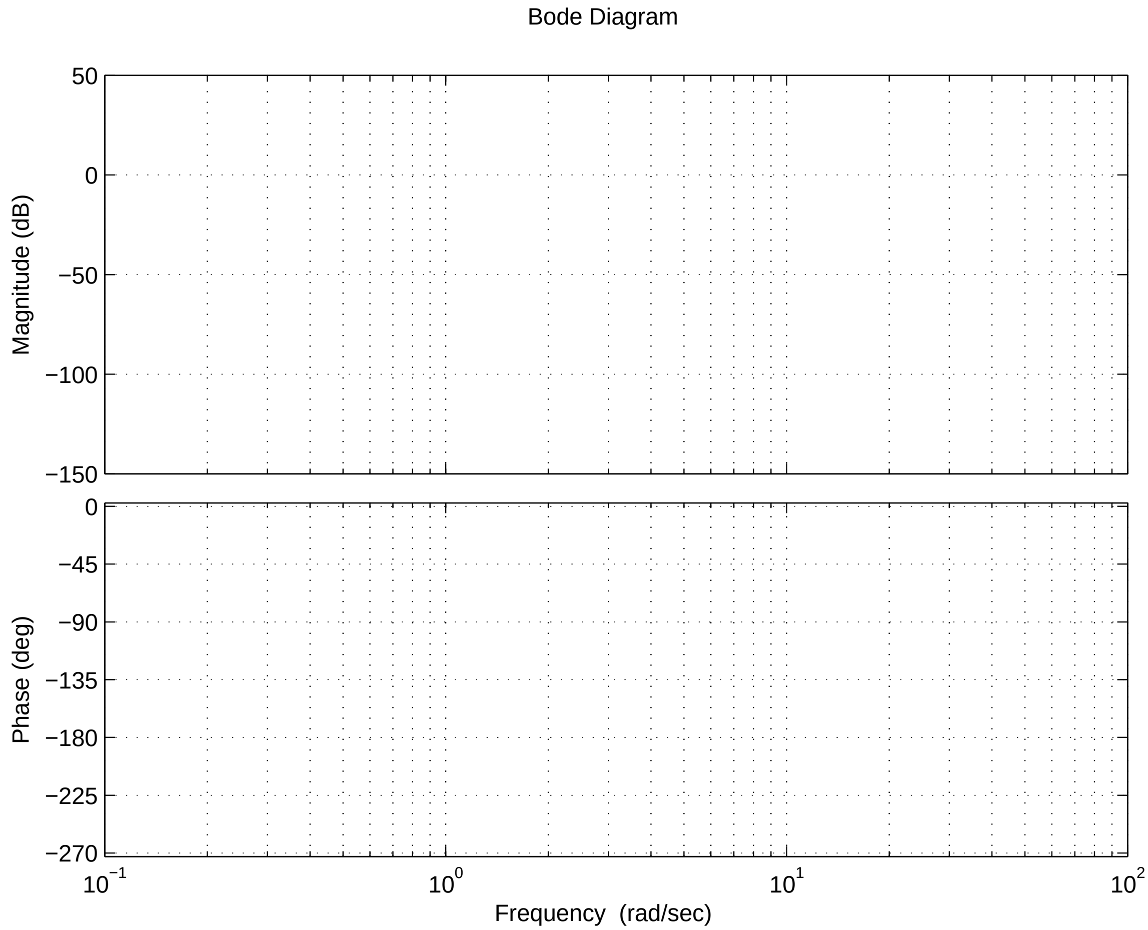

Root Locus
Imaginary Axis
Real Axis
5
4
3
2
1
0
−1
−2
−3
−4
−5
−8
−7
−6
−5
−4
−3
−2
−1
0
1
2

2. Use a Routh array to verify the range of k values in Problem 1 for stability.

3. Sketch the Bode plot for $G(s)$ in Problem 1.

- For clarity, split the transfer functions into terms and call them something like "term 1" and "term 2" and label each of the curves on the plot on this page as corresponding to "term 1," "term 2," *etc.*
- Be sure to indicate the values of all the slopes in the magnitude plot when they are not zero.
- Indicate on the Bode plot the feature that corresponds to the maximum k value for stability that you determined in the first two problems, *i.e.,* how you could have determined the maximum k from the Bode plot. On the blank plots on this page, sketch the individual terms of the plot, and on the next page plot the combined plot.





Bode Diagram
Magnitude (dB)
50
0
−50
−100
−150
Phase (deg)
0
−45
−90
−135
−180
−225
−270
10−1
100
101
102
Frequency (rad/sec)

4. If a lead compensator of the form

$$C(s) = \frac{s+z}{s+p}$$

where $0 < z < p < \infty$ is added to the feedback system in series with $G(s)$, *i.e.*, in the usual way, will it be possible to determine values for the zero and pole such that the closed-loop system is stable for all positive values of k? Explain your answer.