University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
First Exam

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ID Number:______________________________

NAME:______________________________

- Do not start or turn the page until instructed to do so.
- You have 55 minutes to complete this exam.
- This is an closed-book exam. You may consult one 8.5 by 11 inch sheet of notes (both sides) that you prepare.
- You may **not** use a calculator or other electronic device.
- There are three problems. Problems 1 and 2 are worth 40 points each and Problem 3 is worth 20 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

'Tis but a scratch. —The Black Knight

1. Use Laplace transforms to determine the solution to

$$\dot{x} + 2x = \begin{cases} 0, & 0 < t \leq \pi \\ \sin t, & \pi < t \leq 2\pi \\ 0, & t > 2\pi \end{cases}$$

where $x(0) = 1$. A table of Laplace transform pairs for common functions appears at the end of this exam.

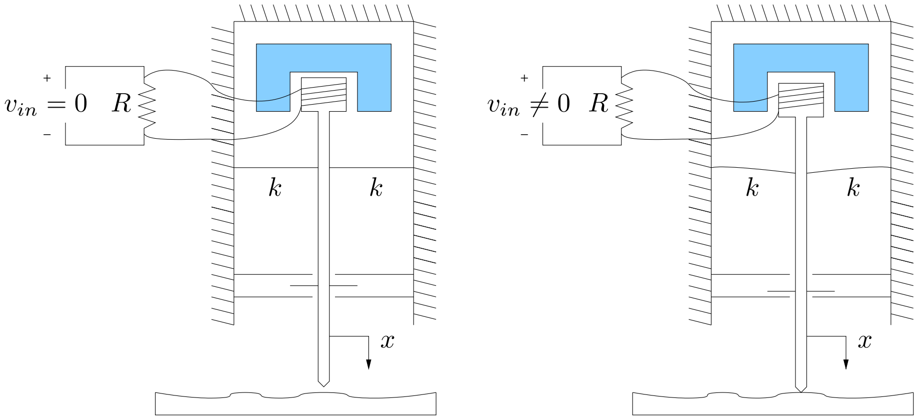

Figure 1. Nanoindeter for Problem 2.

2. It pays to participate in the community of scholarship at Notre Dame because everyone that went to yesterday's AME seminar titled "Size Dependence of Strength of Materials" by Professor Nix from Stanford would have seen the nanoindenter[1] illustrated in Figure 1 and naturally would have wondered what the transfer function for such a system would be.

   The thin pointed plunger with mass m is what makes indentations in the material. The plunger is held in place by two leaf springs, each with spring constant k. A coil of wire is wrapped around the top of the plunger, part of which is inside a permanent magnet. The permanent magnet is the the shaded part. When the voltage in the circuit is not zero, the magnetic field caused by the current in the coil is such that the force on the plunger is proportional to the current, namely, the force due to the magnetic field is given by

$$f(t) = Bi(t)$$

   where $i(t)$ is the current in the circuit and $f(t)$ is the downward force on the plunger. The two leaf springs cause a restoring force. The figure on the left is when the input voltage is zero and the plunger is in the zero position. Similar to d.c. motors and the speaker problem you did, there is a back emf effect when the plunger has a nonzero velocity, and the voltage drop across the coil is given by

$$v_{coil}(t) = k_e \dot{x}(t)$$

   We will not use it for this problem, but the lines below the springs are plates, and the way that the displacement of the plunger is measured is by the change in capacitance when the plates move relative to each other. The figure on the right shows the plunger when it is displaced by a very small amount.

[1]A nanoindenter makes very small (nanoscale) indentations in materials to measure material properties such as hardness and the modulus of elasticity.

(a) Find the transfer function from the input voltage to the position of the plunger, *i.e.,* find $X(s)/V_{in}(s)$ while the plunger is out of contact with the test material.

(b) Use the final value theorem to show that if v_{in} is a step input, if it has a larger magnitude, then the steady-state displacement of the plunger will be larger.

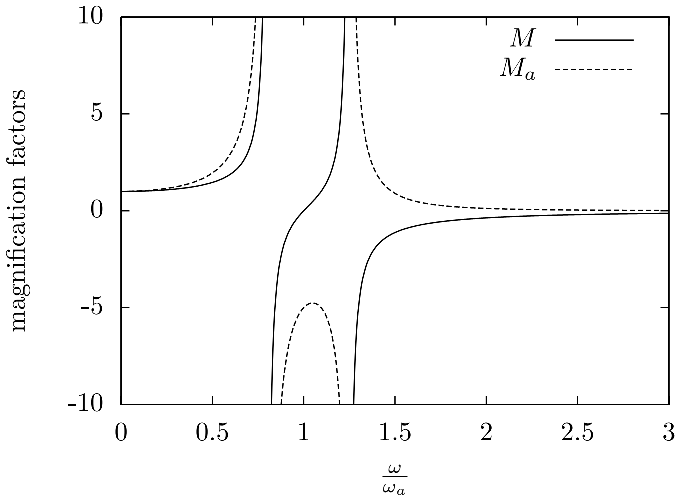

Figure 2. Magnification factors for a vibration absorber with a mass equal to one fifth of the original mass.

3. Figure 2 reproduces Figure 7.10 from the course text and is a plot of the magnitudes of the particular solutions for the two equations

$$\begin{aligned} m\ddot{x} &= -kx - k_a\left(x - x_a\right) + f(t) \\ m_a\ddot{x}_a &= k_a\left(x - x_a\right) \end{aligned}$$

which are the equations of motion for the vibration absorber system illustrated in on the right Figure 3.

It is now 2017 and you work for the Energy Division of General Electric. Their steam generator turbines use vibration absorbers and you may assume that the model they used can be described accurately by the equations and figures given in this problem.

Your boss says there is a problem with the vibration absorber system they have. It works great when the turbine is running at its designed speed. But when it starts or shuts down, during the time when it is speeding up or slowing down it goes through a phase where the whole thing vibrates violently. Your boss is wants to know what is causing that to happen. Explain what is happening. You are not asked to design a solution, but rather just explain, in words, what is happening.

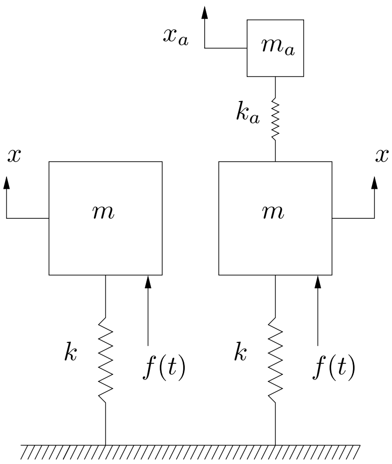

Figure 3. Original system (left) and system with absorber (right).

$f(t), t \geq 0$	$F(s)$
$\delta(t)$	1
$\mathbb{1}(t)$	$1/s$
t	$1/s^2$
t^2	$2!/s^3$
t^3	$3!/s^4$
t^m	$m!/s^{m+1}$
e^{-at}	$1/\left(s+a\right)$
te^{-at}	$1/\left(s+a\right)^2$
$\frac{1}{2!}t^2e^{-at}$	$1/\left(s+a\right)^3$
$\frac{1}{(m-1)!}t^{m-1}e^{-at}$	$1/\left(s+a\right)^m$
$1-e^{-at}$	$a/\left(s\left(s+a\right)\right)$
$\frac{1}{a}(at-1+e^{-at})$	$a/\left(s^2\left(s+a\right)\right)$
$e^{-at}-e^{-bt}$	$\left(b-a\right)/\left(\left(s+a\right)\left(s+b\right)\right)$
$(1-at)e^{-at}$	$s/\left(s+a\right)^2$
$1-e^{-at}(1+at)$	$a^2/\left(s\left(s+a\right)^2\right)$
$be^{-bt}-ae^{-at}$	$\left(\left(b-a\right)s\right)/\left(\left(s+a\right)\left(s+b\right)\right)$
$\sin at$	$a/\left(s^2+a^2\right)$
$\cos at$	$s/\left(s^2+a^2\right)$
$e^{-at}\cos bt$	$\left(s+a\right)/\left(\left(s+a\right)^2+b^2\right)$
$e^{-at}\sin bt$	$b/\left(\left(s+a\right)^2+b^2\right)$
$t\sin at$	$2as/\left(s^2+a^2\right)^2$
$t\cos at$	$\left(s^2-a^2\right)/\left(s^2+a^2\right)^2$
$1-e^{at}\left(\cos bt+\frac{a}{b}\sin bt\right)$	$\left(a^2+b^2\right)/\left(s\left[(s+a)^2+b^2\right]\right)$

Table 1. Table of Laplace transform pairs for functions common in engineering

Name	Time Function	Laplace Transform
Transform pair	$f(t)$	$F(s)$
Superposition	$\alpha f_1(t) + \beta f_2(t)$	$\alpha F_1(s) + \beta F_2(s)$
Differentiation	$\frac{d^m}{dt^m} f(t)$	$s^m F(s) - s^{m-1} f(0) - s^{m-2} \dot{f}(0) - \cdots - s \frac{d^{m-2}}{dt^{m-2}} f(0) - \frac{d^{m-1}}{dt^{m-1}} f(0)$
Time delay ($\tau \geq 0$)	$f(t-\tau)\mathbb{1}(t-\tau)$	$F(s)e^{-s\tau}$
Time scaling	$f(at)$	$F\left(s/a\right)/\lvert a \rvert$
Frequency shift	$e^{-at} f(t)$	$F(s+a)$
Integration	$\int f(\xi) d\xi$	$F(s)/s$
Convolution	$f_1(t) * f_2(t)$	$F_1(s) F_2(s)$
Initial value theorem	$f(0^+)$	$\lim_{s\to\infty} sF(s)$
Final value theorem	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$
Time product	$f_1(t) f_2(t)$	$1/(2\pi i) \int_{c-i\infty}^{c+i\infty} F_1(\xi) F_s(s-\xi) d\xi$
Multiplication by time	$tf(t)$	$-\frac{d}{ds} F(s)$

Table 2. Properties of the Laplace transform