University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
First Exam

B. Goodwine
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ID Number:______________________________

NAME:______________________________

- Do not start or turn the page until instructed to do so.
- You have 75 minutes to complete this exam.
- This is an open book exam. You may consult the course text and your own course notes, but nothing else.
- You may **not** use a calculator or other electronic computing device. If you have the book in electronic form and/or take notes in electronic form, you may consult those.
- There are four problems, each worth 25 points.
- Your grade on this exam will constitute between 0 and 30% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in a Blue Book.
- *Be careful when computing eigenvalues!* If you mistakenly compute distinct real ones and the problem is actually more complicated than that, you can not receive a lot of partial credit because you would will have freed up a lot of time to focus on the other problems.
- If a problem is to compute a solution, you must write out all the numerical components of the solution. Just leaving it as $\xi(t) = e^{At}\xi(0)$ will earn only a point or two.

Perfection is not attainable, but if we chase perfection we can catch excellence.

—Vince Lombardi

1. Determine the general solution to $\dot{\xi}(t) = A\xi(t)$ where

$$A = \begin{bmatrix} -3 & 0 & -1 \\ 0 & -4 & 0 \\ -1 & 0 & -3 \end{bmatrix}.$$

2. Determine the general solution to

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \cos 3t \end{bmatrix}.$$

3. Determine the general solution to

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ -1 & -4 & 0 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}.$$

Problem 4 is on the next page.

Figure 1

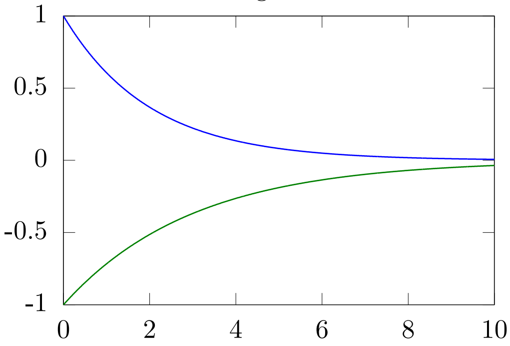


Figure 2

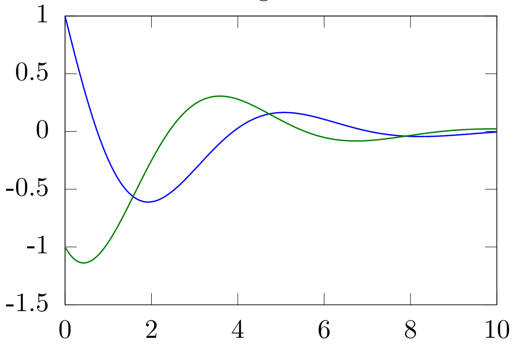


Figure 3

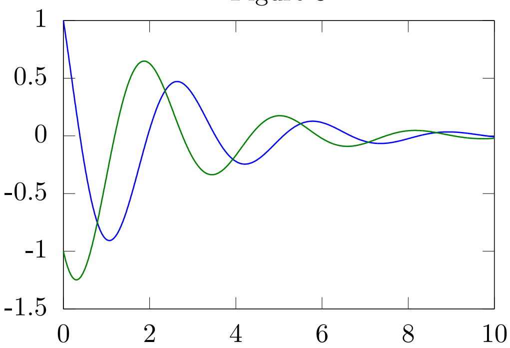
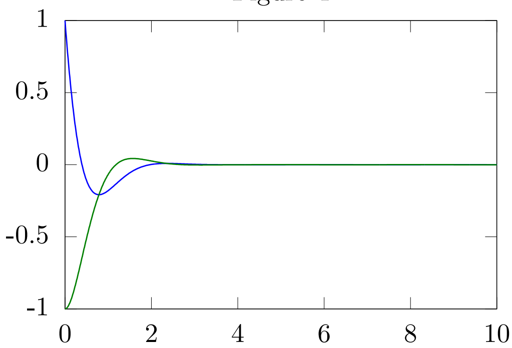


Figure 4



4. Match the following differential equations with the plots of their solutions. The blue curve is $\xi_1(t)$ and the green curve is $\xi_2(t)$.

   Completely justify your answer by whatever computations are necessary. It may be necessary to determine the complete general solution. Or, it may be sufficient to completely justify your answers by only computing the eigenvalues, for example. You will not be penalized for doing extra work, but it will obviously take more time.

   Each problem is $\dot{\xi} = A\xi$ with the same initial conditions.

$$A_1 = \begin{bmatrix} -\frac{1}{2} & 1 \\ -1 & -\frac{1}{3} \end{bmatrix} \quad A_2 = \begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix} \quad A_3 = \begin{bmatrix} -\frac{1}{2} & 2 \\ -2 & -\frac{1}{3} \end{bmatrix} \quad A_4 = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{3} \end{bmatrix}$$