

UNIVERSITY OF NOTRE DAME
Aerospace and Mechanical Engineering

AME 90951: Geometric Nonlinear Control Theory
Homework 1

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1. Assume \mathbb{R}^n has the usual topology and consider

$$S_1 = [0, 2\pi) \subset \mathbb{R}$$
$$S_2 = \{x \in \mathbb{R}^2 \mid \|x\| = 1\} \subset \mathbb{R}^2$$

and equip each with the subset topology.

Define the map

$$f : [0, 2\pi) \rightarrow S_2$$
$$x \mapsto (\cos x, \sin x).$$

Is f a homeomorphism? Justify your answer.¹

2. Let S be a set. Is the identity map from S with the discrete topology to S with the trivial topology continuous? Is it a homeomorphism?
3. Prove that the circle, S^1 is a smooth manifold. Use the stereographic projections illustrated in Figure 1 for the coordinate charts.

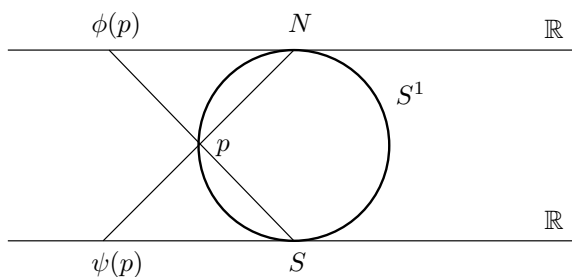


Figure 1: Circle with stereographic projections.

4. Prove Proposition 1.2.1 in the course text.
5. Prove Proposition 1.2.2 in the course text.

¹Problems 1 and 2 are from R Murray from Caltech CDS 102 or 122, circa 1994.