University of Notre Dame
Aerospace and Mechanical Engineering

AME 90951: Geometric Nonlinear Control Theory
Homework 2

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1. Show that the set

$$SO(3) = \left\{ R \in \mathbb{R}^{3\times 3} | RR^T = R^T R = I, \det(R) = +1 \right\}$$

is a manifold with dimension three. Do this by choosing a coordinate chart to parameterize it. Indicate whether the chart is global. If not, indicate how you would determine another chart so that together they cover the whole space. You do not have to find the second chart.

2. Show that if $F : N \to M$ is a diffeomorphism between smooth manifolds N and M, then F_* is an isomorphism between tangent spaces.

3. From Guillemin and Pollack, page 18, number 7.

   (a) Show that $g : \mathbb{R}^1 \to S^1$ defined by $g(t) = (\cos 2\pi t, \sin 2\pi t)$ is a local diffeomorphism.

   (b) Show that $\hat{g} : \mathbb{R}^2 \to S^1 \times S^1$ defined by $\hat{g} = g \times g$ is a local diffeomorphism.

   (c) Show that if L is a line in $\mathbb{R}^2$ then the restriction $\hat{g} : L \to S^1 \times S^1$ is an immersion and if L has irrational slope, then $\hat{g}$ is injective on L.

4. Choose two manifolds that are different and where neither is $\mathbb{R}^n$. Using geometry or some other methods than coordinates, make up two mappings between the manifolds, one of which is smooth and the other if which is not. The mappings do not have to be global. In each case, use a coordinate chart to show if the mapping is smooth or not.

5. Isidori, page 481. Show that the helix example is an embedding.