UNIVERSITY OF NOTRE DAME
Aerospace and Mechanical Engineering

AME 90951: Geometric Nonlinear Control Theory
Homework 4

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1. Consider the ball and beam system illustrated in Figure 1. The variable r describes the position of the ball along the beam, θ is the beam angle, J is the mass moment of inertia of the beam, ρ is the offset of the beam from the point of rotation of the system and everything else should be clear.
   (a) Determine the equations of motion for this system. I think they are close, but not exactly the same as,
   $$\begin{bmatrix} m & m\rho \\ m\rho & J + m\left(r^2+\rho^2\right) \end{bmatrix} \begin{bmatrix} \ddot{r} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} -mr\dot{\theta}^2 \\ 2mr\dot{r}\dot{\theta} \end{bmatrix} + \begin{bmatrix} mg\sin\theta \\ mg\left(r\cos\theta - \rho\sin\theta\right) \end{bmatrix} = \begin{bmatrix} 0 \\ \tau \end{bmatrix}.$$
   (b) If we take the output to be $y = r$, determine is the relative degree and describe the region in which it is defined.
   (c) Does there exist another output function that yields a higher relative degree?
   (d) Analyze the zero dynamics, if any, of the system.
   (e) Design a feedback Linearizing controller and simulate it to try to stabilize the system. Do the simulation results validate your computations?
   (f) Design a controller for tracking and validate your design via simulation.

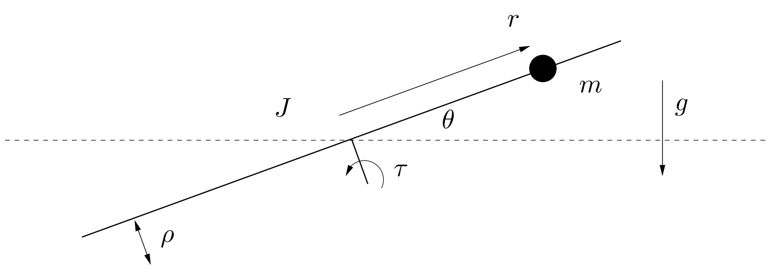

Figure 1: Ball and beam system for Problem 1.

2. Write a computer program to calculate the relative degree of a SIS system. This is a symbolic computation, so a package like Mathematica, Maple or perhaps the symbolic computational components of Matlab may be used.

3. Consider

$$\begin{aligned}\dot{x}_1 &= -x_1 + \frac{2 + x_3^2}{3 + x_3^2} u \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_1 x_3 + 2u.\end{aligned}$$

   (a) Determine the relative degree.
   (b) If needed, determine the stability of the zero dynamics.
   (c) If possible, design a stabilizing feedback controller.

4. Consider the *controlled van der Pol oscillator*

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + \alpha \left(1 - x_1^2\right) x_2 + u.\end{aligned}$$

   If the output is taken to be $y = x_1$ what is the relative degree? Is the system full state feedback linearizable? If so, design a stabilizing controller. If not, analyze the zero dynamics and if possible design a stabilizing controller. In all cases, does the sign of α matter? Verify your results with numerical simulations.

5. Consider a very generic second order system

$$\ddot{x} + f(\dot{x}, x) = u.$$

   If $y = x$ what is the relative degree?

6. Consider

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= u \\ \dot{x}_3 &= x_1 + x_2^2.\end{aligned}$$

   Does there exists an output function $y = h(x)$ which renders the system full state feedback linearizable? If so, find it and design a stabilizing controller.

7. Consider

$$\begin{aligned}\dot{x}_1 &= x_3(1 + x_2) \\ \dot{x}_2 &= x_1 + (1 + x_2)u \\ \dot{x}_3 &= x_2(1 + x_1) - x_3 u.\end{aligned}$$

   Is this system full-state feedback linearizable? If so, find an output function verifying that fact. Design a tracking controller and verify the efficacy of the controller via simulation.