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Cyber-Physical Systems Design Using Dissipativity and Symmetry

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Cyber-Physical Systems (CPS)

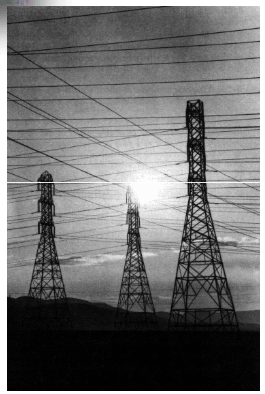
-As computers become ever-faster and communication bandwidth ever-cheaper, computing and communication capabilities will be embedded in all types of objects and structures in the physical environment.

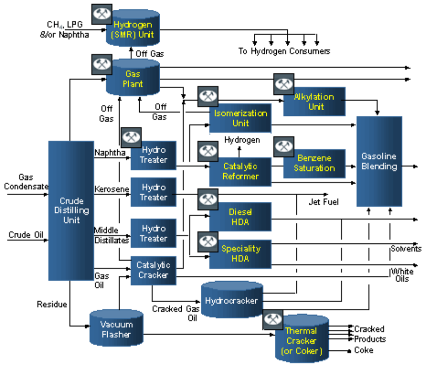
-***Cyber-physical systems (CPS)*** are physical, biological and engineered systems whose ***operations are monitored, coordinated,*** **controlled** ***and integrated by a computing and communication core***.

-This intimate coupling between the cyber and physical will be manifested from the nano-world to large-scale wide-area systems of systems. And at multiple time-scales.

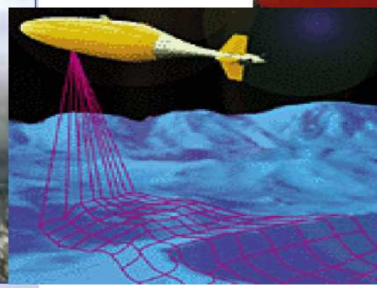
-Applications with enormous societal impact and economic benefit will be created. Cyber-physical systems will transform how we interact with the physical world just like the Internet transformed how we interact with one another.

-**We should care about CPS because our lives depend on them**





UAVs operating together



Technological and Economic Drivers

- The decreasing cost of computation, networking, and sensing.
- A variety of social and economic forces will require more efficient use of national infrastructures.
- Environmental pressures will mandate the rapid introduction of technologies to improve energy efficiency and reduce pollution.
- The need to make more efficient use of health care systems, ranging from facilities to medical data and information.

What is a CPS?
Can we recognize a CPS when we see it?

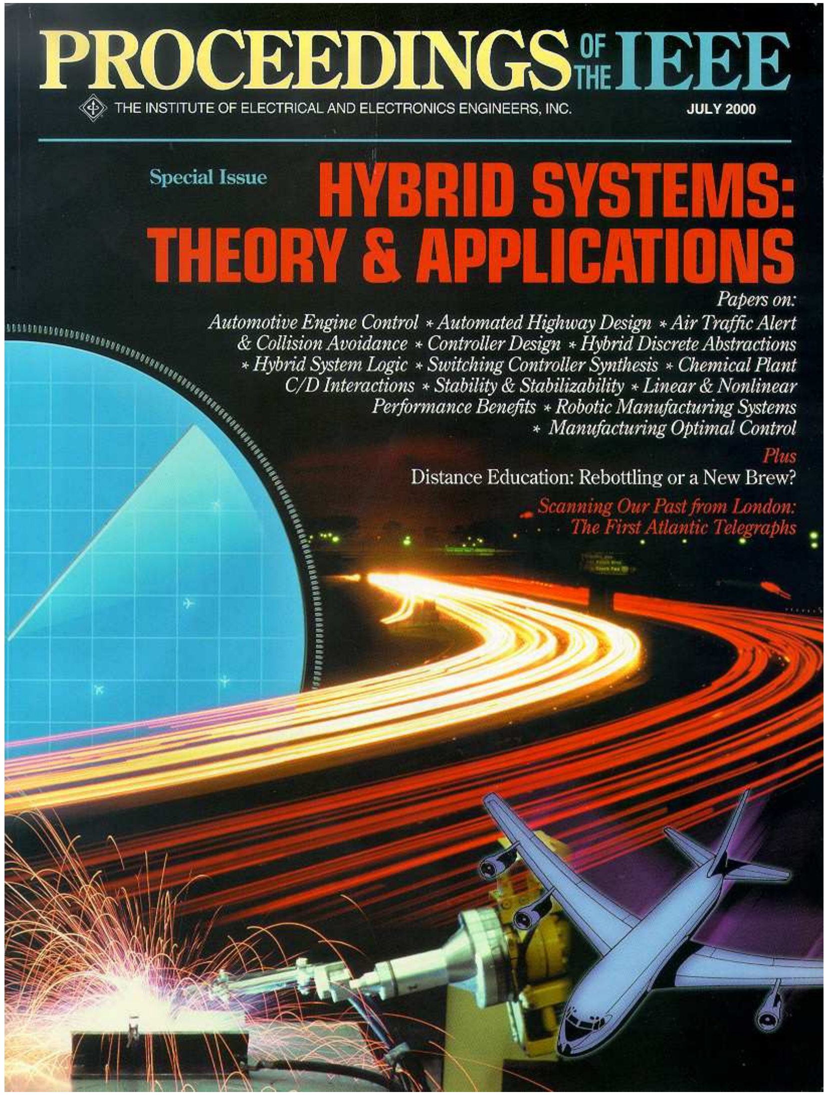
- NIT interaction with physical world.
- Digital control in chemical processes half a century old.??

- What is different here is the tight integration of the cyber and the physical parts.

- The specifications play a central role. When demanding, the cyber and physical should coordinate and orchestrate their actions and reactions to achieve desired goals.

What is a CPS?
Can we recognize a CPS when we see it?

- Example from hybrid systems. Train and gate. To minimize time gate is down, stopping cars from crossing, is obtained when the continuous dynamics of the train and the gate are taken into account.
- When there is no heat or energy issue or shared resource issues, there is no reason to worry about reducing the clock speed of the digital device when the control algorithm does not need it, do cross layer design, worry about demanding timing issues in implementing the algorithm
- **So the same system may be regarded as a CPS or not.**


PROCEEDINGS OF THE IEEE
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Special Issue
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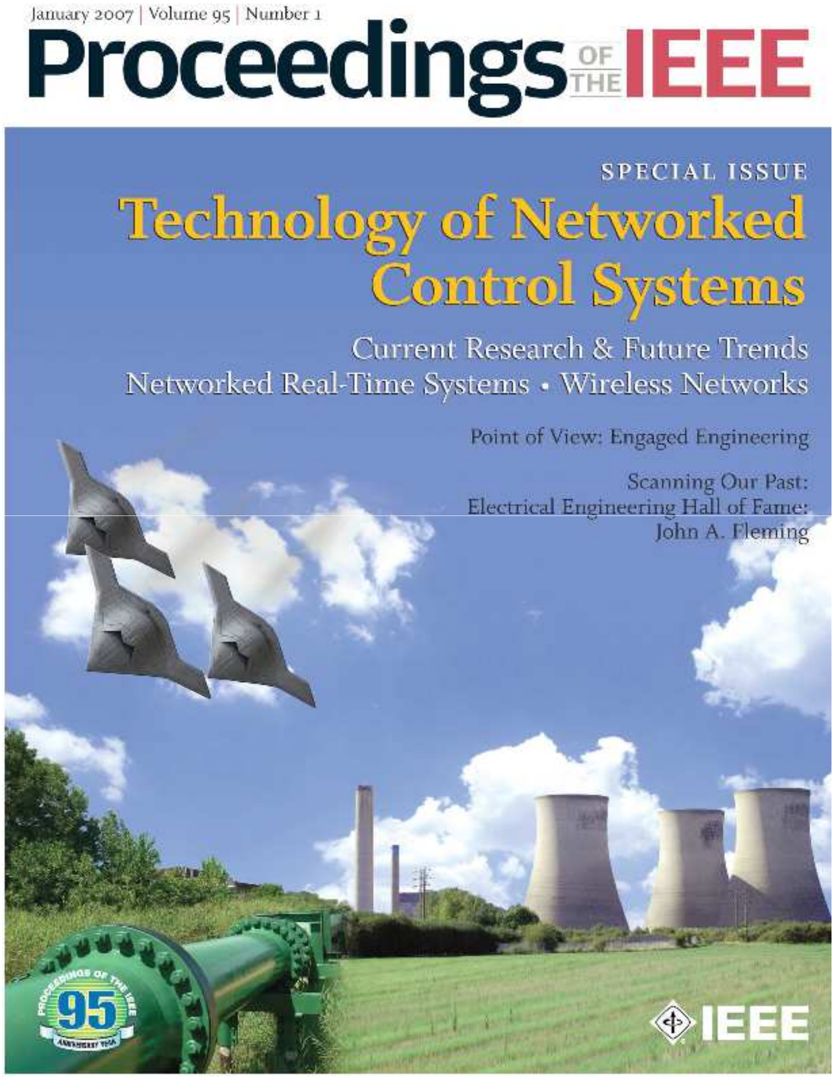
January 2007 | Volume 95 | Number 1
Proceedings OF THE IEEE
SPECIAL ISSUE
Technology of Networked Control Systems
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Networked Real-Time Systems • Wireless Networks
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Scanning Our Past: Electrical Engineering Hall of Fame: John A. Fleming
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Connections & Personal Motivation

Linear Feedback Systems

– Polynomial Matrix Descriptions

Autonomous Intelligent Control

- Defining Intelligent Control - Hierarchies - Degrees of Autonomy
- To DES (PN), Hybrid Systems, Networked Control Systems (MB)
- To Cyber Physical Systems

Quest for Autonomy

- Autonomy wrt …. Human in the loop.

The Power of Feedback

– Feedback Transcends Models

PCAST Report

Leadership Under Challenge:
Information Technology R&D in a Competitive World
An Assessment of the Federal Networking and Information Technology R&D Program
President's Council of Advisors on Science and Technology
August 2007

New Directions in Networking and Information Technology (NIT)

Recommendation: No 1 Funding Priority:
NIT Systems Connected with the Physical World

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CPS Challenges

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CPS Characteristics

What cyber physical systems have as defining characteristics:

- Cyber capability (i.e. networking and computational capability) in every physical component
- They are networked at multiple and extreme scales
- They are complex at multiple temporal and spatial scales.
- They are dynamically reorganizing and reconfiguring
- Control loops are closed at each spatial and temporal scale. Maybe human in the loop.
- Operation needs to be dependable and certifiable in certain cases
- Computation/information processing and physical processes are so tightly integrated that it is not possible to identify whether behavioral attributes are the result of computations (computer programs), physical laws, or both working together.

CPS Issues

There is a set of pervasive underlying problems for CPS not solved by current technologies:

- How to build predictable real time, networked CPS at all scales?

- How to build and manage high-confidence, secure, dynamically-configured systems?

- How to organize and assure interoperability?

- How to avoid cascading failure?

- How to formulate an evidential (synthetic and analytic) basis for trusted systems? Certified.

How could these issues be addressed?

- Assuming exact knowledge of the components and their interconnections may not be reasonable.
- Dynamic change. The physical part may cause the CPS to change. Links disappear. Modules stop operating. These are to be expected when we are interested in the whole life cycle of the system.
- If the system was safe, verified to be safe, can we guarantee that it will still be? Can we do something about it? Is it resilient? High autonomy.
- If secure originally can we still guarantee that property?
- Connections to linear programming, optimization. Simplex and sensitivity analysis.

How could these issues be addressed?

- Perhaps it is more reasonable to aim for staying in operating regions. Operating envelope.
- Flight envelope. The pilot is not allowed to take certain actions that may stall the aircraft (Airbus). Flight envelope.

- In DES supervisory control actions are allowed or not allowed to occur and so behavior is restricted
- Lyapunov stability implies that the states are bounded-asymptotic stability implies that the state will also go to the origin as time goes to infinity. Restrictions on behavior.

- Feedback interconnection of stable systems may not be stable. Switching among stable systems may lead to unstable systems.

- Is there any similar, energy like concept where guarantees can be given about properties in, say, feedback configurations?

Approaches to Meet the Challenges

Passivity and Symmetry in CPS

- **In CPS, heterogeneity causes major challenges. In addition network uncertainties-time-varying delays, data rate limitations, packet losses.**

-**Need to guarantee properties of networks of heterogeneous systems that dynamically expand and contract.**

-**Need results that offer insight on how to do synthesis – how to grow the system to preserve certain properties.**

- **We impose passivity constraints on the components and use wave variables, and the design becomes insensitive to network effects. Stability and performance.**

- **Symmetry.**

Thanks to:

- **Han Yu, Mike McCourt, Po Wu, Feng Zhu, Meng Xia**
- **Eloy Garcia, Yue Wang, Getachew Befekadu**
- **Vijay Gupta, Bill Goodwine**

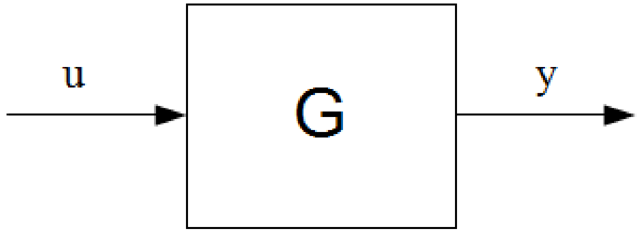
- ***NSF CPS Large: Science of Integration for CPS, Vanderbilt, Maryland, Notre Dame, GM R&D***

Background on Passivity

Definition of Passivity in Continuous-time

- Consider a continuous-time nonlinear dynamical system

$$\dot{x} = f(x,u)$$
$$y = h(x,u).$$



- This system is *passive* if there exists a continuous storage function *V(x)* ≥ *0 (for all x)* such that

$$\int_{t_1}^{t_2} u^T(t)y(t)dt + V(x(t_1)) \geq V(x(t_2))$$

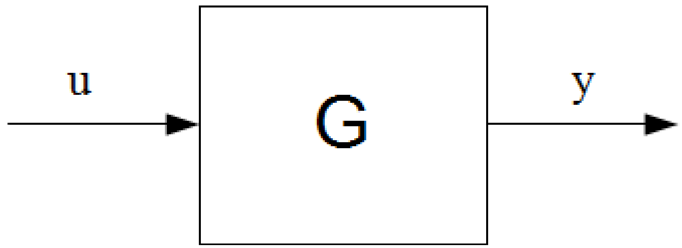
for all $t_2 \geq t_1$ and input $u(t) \in U$.

- When V(x) is continuously differentiable, it can be written as:

$$u^T(t)y(t) \geq \dot{V}(x(t))$$

Passivity in Discrete-time

- Passivity can also be defined for discrete-time systems. Consider a nonlinear discrete time system



$$x(k+1) = f(x(k), u(k))$$
$$y(k) = h(x(k), u(k)).$$

- This system is *passive* if there exists a continuous storage function $V(x) \geq 0$ such that

$$\sum_{k=k_1}^{k_2} u^T(k) y(k) + V(x(k_1)) \geq V(x(k_2))$$

for all k_1, k_2 and all inputs $u(k) \in U$.

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Extended Definitions of Passivity

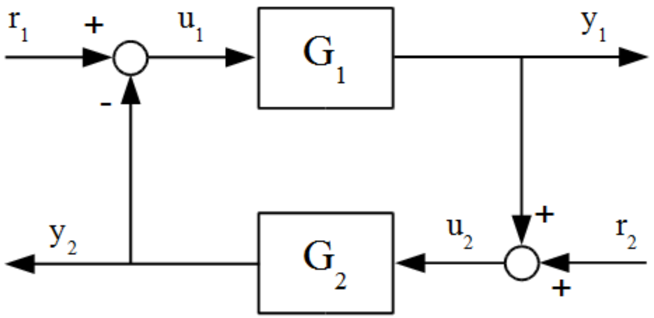
Passive	$u^T y \geq \dot{V}(x)$
Lossless	$u^T y = \dot{V}(x)$
Strictly Passive	$u^T y \geq \dot{V}(x) + \psi(x)$
Strictly Output Passive	$u^T y \geq \dot{V}(x) + \varepsilon y^T y$
Strictly Input Passive	$u^T y \geq \dot{V}(x) + \delta u^T u$

- Note that *V(x)* and *Ψ(x)* are positive definite and continuously differentiable. The constants ε and δ are positive. These equations hold for all times, inputs, and states.

Interconnections of Passive Systems

- One of the strengths of passivity is when systems are interconnected. Passive systems are stable and passivity is preserved in many practical interconnections.

- For example, the negative feedback interconnection of two passive systems is passive.



- If $u_1 \to y_1$ and $u_2 \to y_2$ are passive then the mapping $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \to y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ is passive

- Note: the other internal mappings ($u_1 \to y_2$ and $u_2 \to y_1$) will be stable but may not be passive

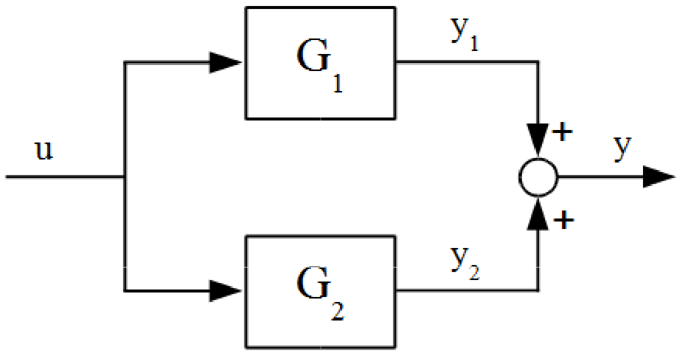
Stability of Passive Systems

- Strictly passive systems ($\psi(x)>0$) are asymptotically stable
- Output strictly passive systems ($\delta>0$) are L_2 stable
- The following results hold in feedback
  - Two passive systems → passive and stable loop
  - Passive system and a strictly passive system → asymptotically stable loop
  - Two output strictly passive systems → L_2 stable loop
  - Two input strictly passive systems ($\varepsilon>0$) → L_2 stable loop

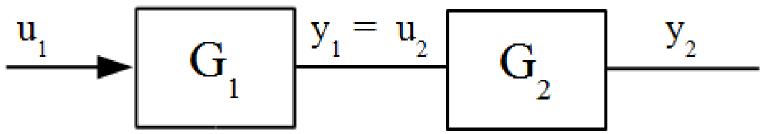
$$u^T y \geq \dot{V}(x) + \varepsilon u^T u$$

Other Interconnections

- The parallel interconnection of two passive systems is still passive



- However, this isn't true for the series connection of two systems



- For example, the series connection of any two systems that have 90° of phase shift have a combined phase shift of 180°

Dissipativity, conic systems, and passivity indices

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Definition of Dissipativity (CT)

- This concept generalizes passivity to allow for an arbitrary energy supply rate *ω(u,y).*
- A system is *dissipative* with respect to supply rate *ω(u,y)* if there exists a continuous storage function $V(x) \geq 0$ such that

$$\int_{t_1}^{t_2} \omega(u,y)dt \geq V(x(t_2)) - V(x(t_1))$$

for all t_1, t_2 and the input $u(t) \in U$.

- A special case of dissipativity is the QSR definition where the energy supply rate takes the following form:

$$\omega(u,y) = y^T Q y + 2y^T S u + u^T R u.$$

- *QSR dissipative systems are L_2 stable when $Q<0$*

QSR Dissipativity (CT)

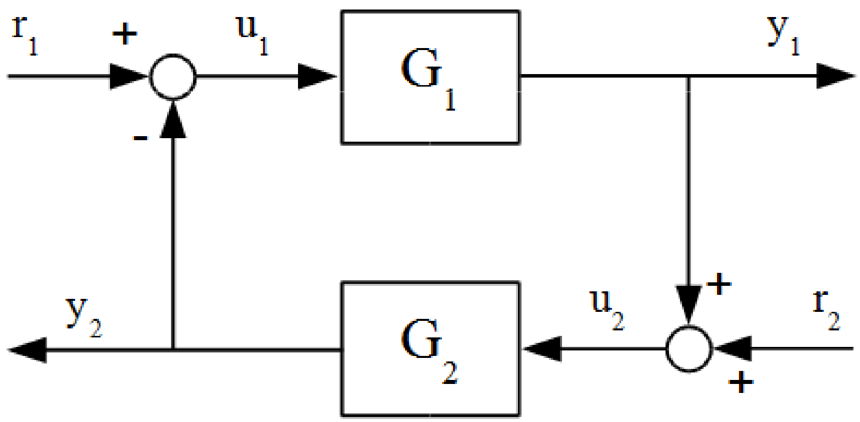
- Consider the feedback interconnection of G_1 and G_2
  - G_1 is QSR dissipative with Q_1, S_1, R_1
  - G_2 is QSR dissipative with Q_2, S_2, R_2

- The feedback interconnection

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \to y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

is stable if

$$\tilde{Q} = \begin{bmatrix} Q_1 + R_2 & S_1 - S_2^T \\ S_1^T - S_2 & Q_2 + R_1 \end{bmatrix} < 0$$



- Other mappings ($r_1 \to y_2$ and $r_2 \to y_1$) are stable but may not be passive
- Large scale sytems (with multiple feedback connections) can be analyzed using QSR dissipativity to show stability of the entire system

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Definition of Dissipativity (DT)

- The concept of dissipativity applies to discrete time systems for an arbitrary supply rate *ω(u,y).*

- A system is *dissipative* with respect to supply rate *ω(u,y)* if there exists a continuous storage function $V(x) \geq 0$ such that

$$\sum_{k=k_1}^{k_2} \omega(u,y) \geq V(x(k_2)) - V(x(k_1))$$

  for all k_1, k_2 and the input $u(k) \in U$.

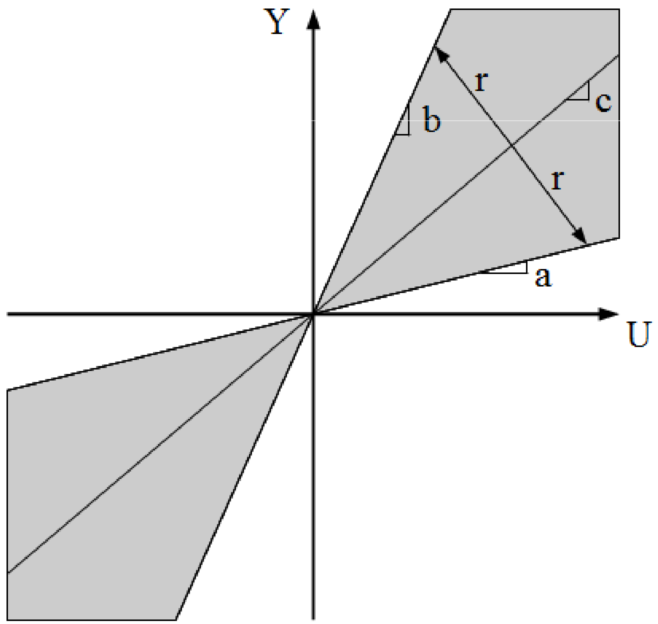
- A special case of dissipativity is the QSR definition where the energy supply rate takes the following form:

$$\omega(u,y) = y^T Q y + 2 y^T S u + u^T R u.$$

- Dissipative DT systems are stable when $Q<0$

Conic Systems

- A conic system is one whose input-output behavior is constrained to lie in a cone of the *U x Y* inner product space



- A system is conic if the following dissipative inequality holds for all $t_2 \geq t_1$

$$\int_{t_1}^{t_2} \left[(1+\frac{a}{b})u^T y - a u^T u - \frac{1}{b} y^T y \right] dt$$

$$\geq V(x(t_2)) - V(x(t_1))$$

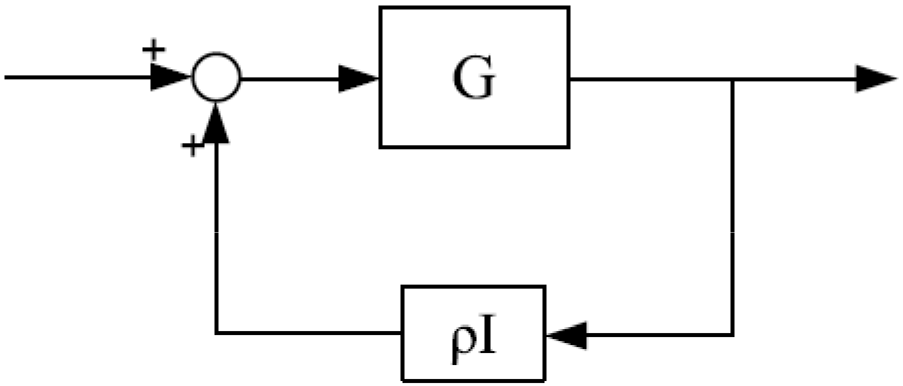
Passivity Indices

- Conic systems and passivity indices capture similar information about a system

- A passivity index measures the level of passivity in a system
- Two indices are required to characterize the level of passivity in a system
  1. The first measures the level of stability of a system
  2. The second measures the extent of the minimum phase property in a system
- They are independent in the sense that knowing one provides no information about the other
- Each has a simple physical interpretation

Output Feedback Passivity Index

The output feedback passivity index (OFP) is the largest gain that can be put in positive feedback with a system such that the interconnected system is passive.

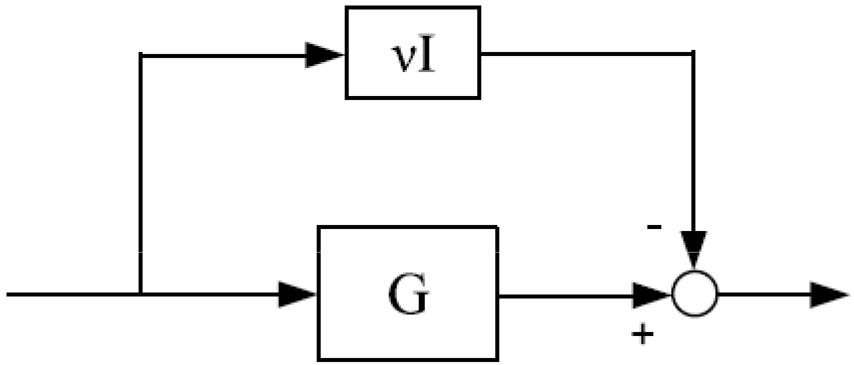


Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y\,dt \geq V(x(t_2)) - V(x(t_1)) + \rho \int_{t_1}^{t_2} y^T y\,dt$$

Input Feed-Forward Passivity Index

The input feed-forward passivity index (IFP) is the largest gain that can be put in a negative parallel interconnection with a system such that the interconnected system is passive.

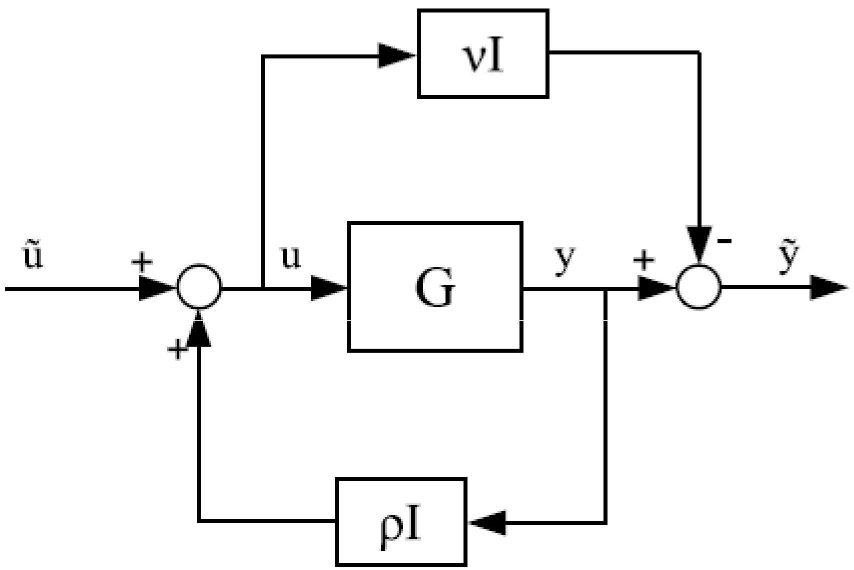


Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y\,dt \geq V(x(t_2)) - V(x(t_1)) + \nu \int_{t_1}^{t_2} u^T u\,dt$$

Simultaneous Indices

When applying both indices the physical interpretation as in the block diagram

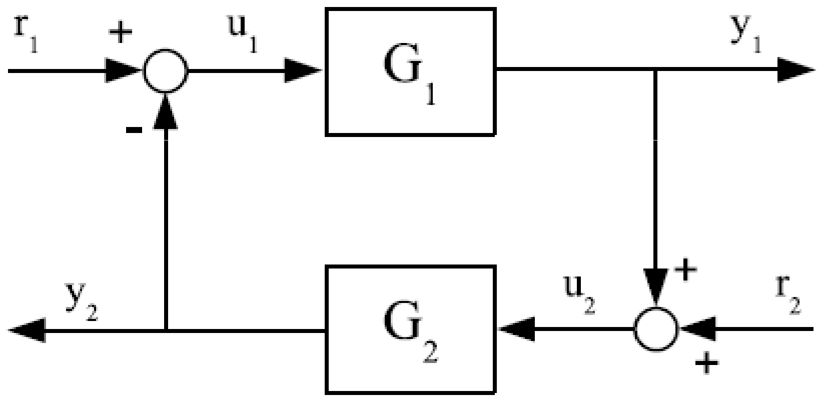


Equivalent to the following dissipative inequality holding for G

$$(1+\rho\nu)\int_{t_1}^{t_2} u^T y\,dt \geq V(x(t_2))-V(x(t_1))+\rho\int_{t_1}^{t_2} y^T y\,dt+\nu\int_{t_1}^{t_2} u^T u\,dt$$

Stability

We can assess the stability of an interconnection using the indices for G_1 and G_2



G_1 has indices ρ_1 and ν_1

G_2 has indices ρ_2 and ν_2

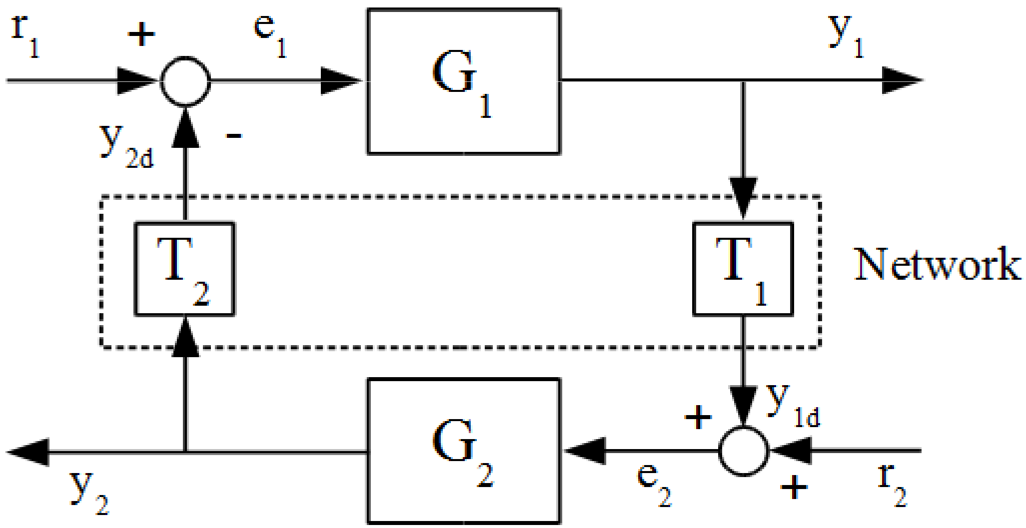
The interconnection is $\mathcal{L}_2$ stable if the following matrix is positive definite

$$\begin{bmatrix} (\rho_1 + \nu_2)I & \frac{1}{2}(\rho_2\nu_2 - \rho_1\nu_1)I \\ \frac{1}{2}(\rho_2\nu_2 - \rho_1\nu_1)I & (\rho_2 + \nu_1)I \end{bmatrix} > 0$$

Networked passive systems

Networked Systems

- Motivating Problem: The feedback interconnection of two passive systems is passive and stable. However, when the two are interconnected over a delayed network the result is not passive so stability is no longer guaranteed. How do we recover stability?



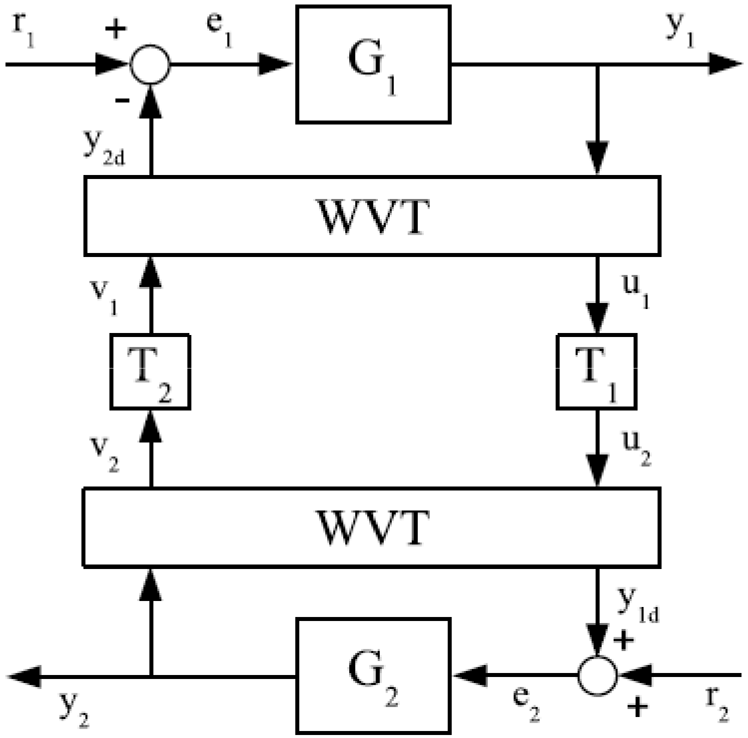
The systems G_1 and G_2 are interconnected over a network with time delays T_1 and T_2

Stability of Networked Passive Systems

- One solution to interconnecting passive systems over a delayed network is to add an interface between the systems and the network

- The wave variable transformation forces the interconnection to meet the small gain theorem. Stability is guaranteed for arbitrarily large time delays

- The WVT is defined below



$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_1 \\ y_{2d} \end{bmatrix} \qquad \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_{1d} \\ y_2 \end{bmatrix}$$

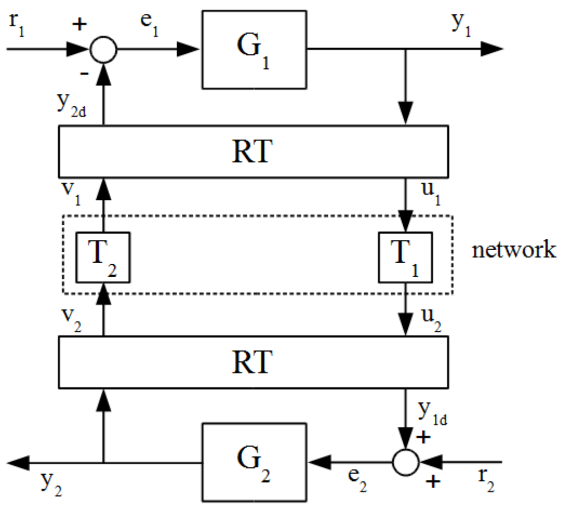
Stability of Networked Conic Systems

- This approach can be generalized to conic systems with a new transformation
- The rotational transformation (RT) can turn any conic system into an L_2 stable system with an appropriate rotation θ

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} cos(\theta)I & sin(\theta)I \\ -sin(\theta)I & cos(\theta)I \end{bmatrix} \begin{bmatrix} y_{2d} \\ y_1 \end{bmatrix}$$

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} cos(\theta)I & -sin(\theta)I \\ sin(\theta)I & cos(\theta)I \end{bmatrix} \begin{bmatrix} y_2 \\ y_{1d} \end{bmatrix}$$

- This approach decouples the control design from the network design
- First, G_2 can be designed using traditional conic systems theory
- The upper RT pre-stabilizes G_1 (in the L_2 sense) to tolerate network delays
- The lower RT inverts the rotation to preserve the conic sector of G_1



Computational methods for showing passivity and dissipativity

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LMI Methods – Passivity

- There are computational methods to find storage functions. For LTI passive systems, can always assume there exists a quadratic storage function

$$V(x) = \frac{1}{2} x^T P x \quad \text{where} \quad P = P^T > 0.$$

- For continuous-time system this leads to the following LMI

$$\begin{bmatrix} A^T P + PA & PB - C^T \\ B^T P - C & -D - D^T \end{bmatrix} \le 0$$

- In discrete-time the LMI becomes the following

$$\begin{bmatrix} A^T PA - P & A^T PB - C^T \\ B^T PA - C & B^T PB - D - D^T \end{bmatrix} \le 0$$

LMI Methods – QSR Dissipativity

- The same can be done to demonstrate that an LTI system is QSR dissipative. Once again, a quadratic storage function is used

$$V(x) = \frac{1}{2} x^T P x \quad \text{where} \quad P = P^T > 0.$$

- For continuous-time system this leads to the following LMI

$$\begin{bmatrix} A^T P + PA - C^T QC & PB - C^T QD - C^T S \\ B^T P - D^T QC - S^T C & -D^T QD - S^T D - D^T S - R \end{bmatrix} \leq 0$$

- In discrete-time the LMI becomes the following

$$\begin{bmatrix} A^T PA - P - C^T QC & A^T PB - C^T QD - C^T S \\ B^T PA - D^T QC - S^T C & B^T PB - D^T QD - S^T D - D^T S - R \end{bmatrix} \leq 0$$

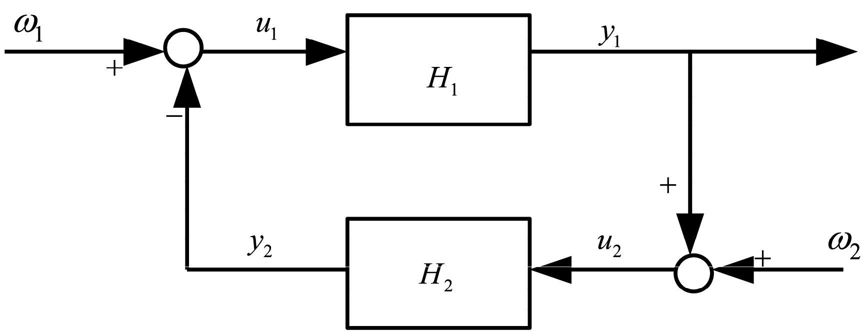
Passivity and CPS. Some Recent Research.

1. A Passivity Measure Of Systems In Cascade Based On Passivity Indices
2. Passivity-Based Output Synchronization With Application To Output Synchronization of Networked Euler-Lagrange Systems Subject to Nonholonomic Constraints
3. Event-Triggered Output Feedback Control for Networked Control Systems using Passivity
4. Output Synchronization of Passive Systems with Event-Driven Communication
5. Quantized Output Synchronization of Networked Passive Systems with Event-driven Communication

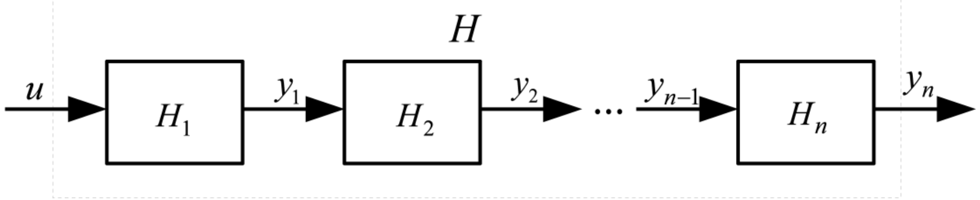
Passivity of systems in series

A Passivity Measure Of Systems In Cascade Based On Passivity Indices (Yu & Antsaklis, CDC10)

➢Problem Statement



- The negative feedback interconnection of two passive systems is still passive.



- Will the cascade interconnection of several passive systems still be passive?

H.Yu and Panos J. Antsaklis, “A Passivity Measure Of Systems In Cascade Based On The Analysis Of Passivity Indices”, Proceedings of IEEE Conference on Decision and Control, Atlanta, Georgia, USA, December 15-17, 2010.

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A Passivity Measure Of Systems In Cascade Based On Passivity Indices (Yu & Antsaklis, CDC10) : main results

Each H_i is IF-OFP(ν_i,ρ_i) such that $\dot{V}_i \leq u_i^T y_i - \rho_i y_i^T y_i - \nu_i u_i^T u_i$ with $\nu_i, \rho_i \in \mathbb{R}$. If for some $\hat{\rho}, \hat{\nu} \in \mathbb{R}$ and

$$A = \begin{bmatrix} -\nu_1 + \hat{\nu} & \frac{1}{2} & 0 & \cdots & -\frac{1}{2} \\ \frac{1}{2} & -\nu_2 - \rho_1 & \frac{1}{2} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{2} & -\nu_n - \rho_{n-1} & \frac{1}{2} \\ -\frac{1}{2} & 0 & \cdots & \frac{1}{2} & -\rho_n + \hat{\rho} \end{bmatrix},$$

such that $-A$ is quasi-dominant,

then cascade system admits a storage function of the form

$$V = \sum_{i=1}^{n} d_i V_i, \ d_i > 0, \ n \geq 2$$

and the cascade interconnection is IF-OFP($\hat{\nu}$,$\hat{\rho}$).

Event-triggered control of passive systems

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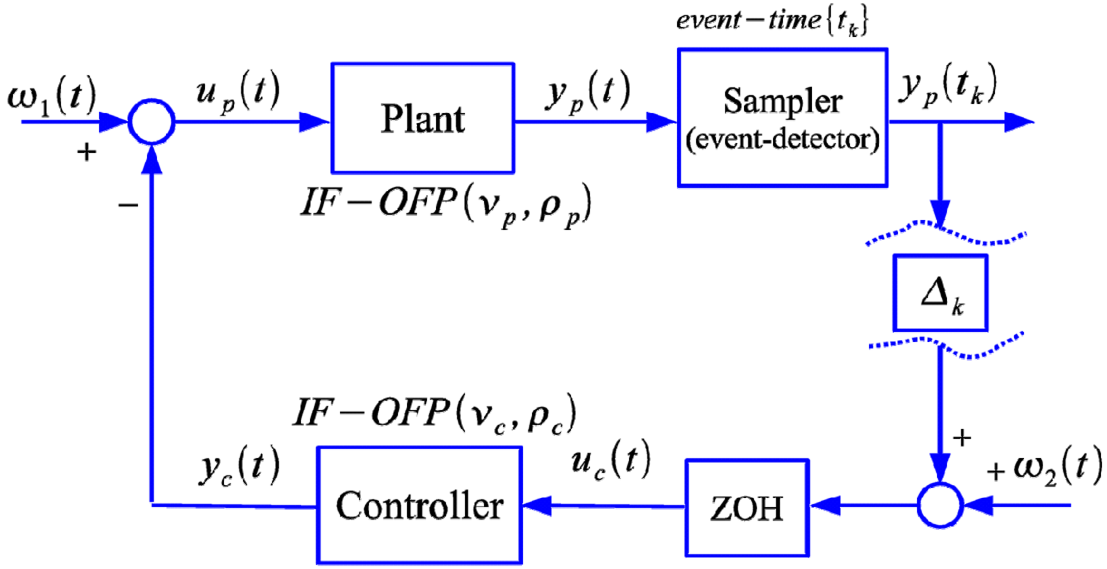
Event-Triggered Control for Networked Systems using Passivity

- Event triggered control is used to reduce communication in networked control systems
- This approach studies event-triggered control from an input-output perspective instead of a state based model
- The implementation of our event-triggered control strategy does not impose constraints on the maximal admissible network induced delays

[Yu and Antsaklis 2011 CDC]

Event-Triggered Network

- When an event occurs the sampled output $y_p(t_k)$ is sent to the networked controller
- Events occur when a triggering condition is met



- One such triggering condition is to maintain closed loop stability
- This can be done by using passivity indices of the plant (ρ_p and v_p) and controller (ρ_c and v_c)

Triggering Condition

- Closed loop stability can be maintained using the passivity indices of the plant (ρ_p and ν_p) and controller (ρ_c and ν_c)
- For now, assume the network delay is zero
- An appropriate triggering condition can be found according to

$$\|\tilde{e}_p(t)\|_2 = \frac{\delta}{\zeta}\left[\sqrt{\beta(\rho_p + \nu_c) + \frac{\nu_c^2}{\zeta^2}} - \frac{|\nu_c|}{\zeta}\right]\|y_p(t)\|_2, \ \forall t \geq 0,$$

$$\text{where} \quad \tilde{e}_p(t) = y_p(t) - y_p(t_k), \quad \text{for} \quad t \in [t_k, t_{k+1}],$$

$$\zeta = \left[\frac{1}{4\alpha(\nu_p + \rho_c)} + |\nu_c| - \nu_c\right]^{\frac{1}{2}},$$

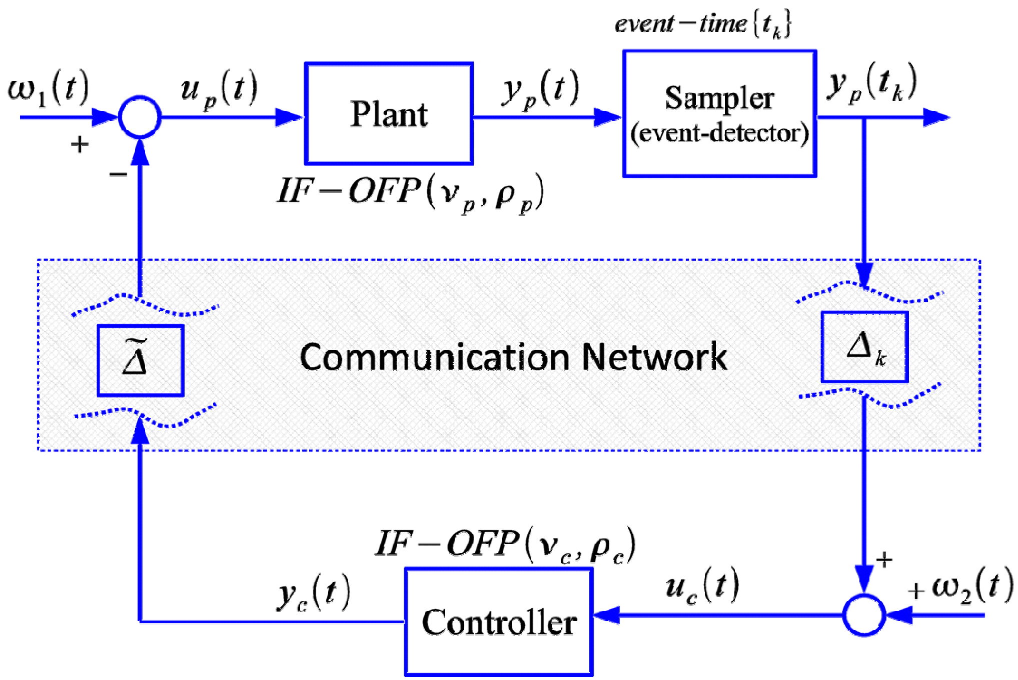
where α, β, and δ are constants between 0 and 1.

- Then the closed loop system with the network is L_2 stable

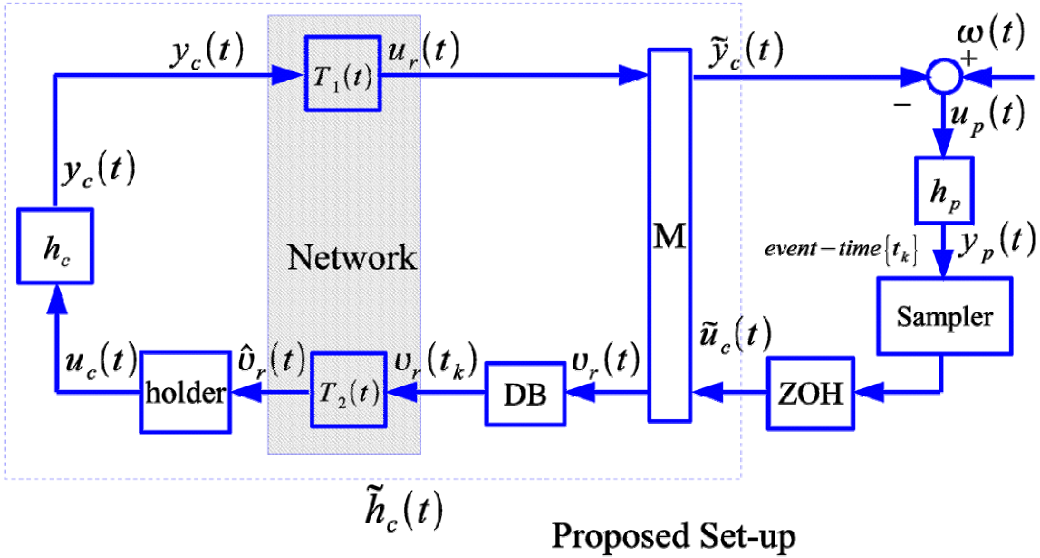
[Yu & Antsaklis 2011 CDC]

Delay Over the Network

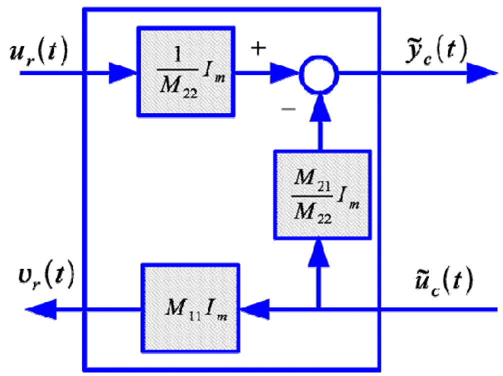
Many control architectures have communication delays on both directions of the network



Compensating for Delays



Proposed Set-up



Local controller M

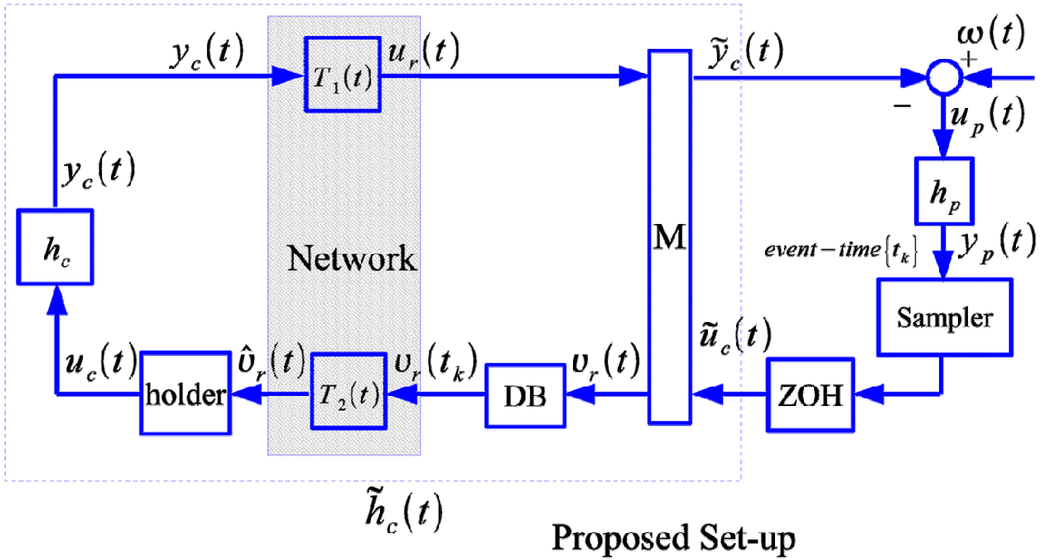
The proposed design method decouples the design of a controller from the design of the network with delays

The network controller h_c has been designed for stability

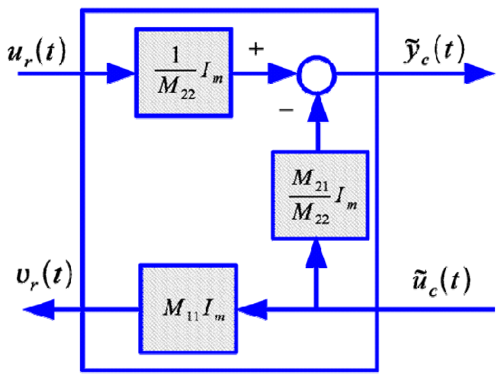
$$0 < \rho_c + \nu_p < \infty, \quad 0 < \rho_p + \nu_c < \infty$$

The transformation M can be designed to compensate for delays

Defining an Appropriate Transformation



Proposed Set-up



Local controller M

$$M_{11}^2 = \frac{\frac{1}{4\rho_c} - \nu_c}{\frac{1}{2\rho_c} + |\nu_c|}, \quad M_{21}^2 = \frac{1}{2(1-D_1)\rho_c^2}$$

$$M_{22}^2 = \frac{2}{1-D_1}, \quad M_{21}M_{22} < 0,$$

$$\begin{bmatrix} \upsilon_r(t) \\ u_r(t) \end{bmatrix} = M = \begin{bmatrix} M_{11}I_m & 0 \\ M_{21}I_m & M_{22}I_m \end{bmatrix} \begin{bmatrix} \tilde{u}_c(t) \\ \tilde{y}_c(t) \end{bmatrix}$$

[Yu & Antsaklis 2011 CDC]

Output synchronization of passive systems using event-driven communication

Traditional Output Synchronization

- Output synchronization is the agreement of a group agents on a particular output value

- Communication is modeled using graph theory – the graph can change over time but it is assumed that the communication graph is at least weakly connected

- Each agent updates its own output value by using the output values of its neighboring agents

$$u_i(t) = \sum_{j\in\mathcal{N}_i} K\left[y_j(t) - y_i(t)\right], \quad i = 1, 2, \ldots, N$$

- Previous work has shown that the group of agents will synchronize using this update law when communication is periodic

Output Synchronization Using Event-Driven Communication

- Using event-driven communication, the update law holds previous values between update times

$$u_i(t) = \sum_{j \in \mathcal{N}_i} a\left[\widehat{y}_j(t) - \widehat{y}_i(t)\right]$$

$$\widehat{y}_j(t) = y_j(t^j_{k'}), \text{ for } t \in [t^j_{k'}, t^j_{k'+1}] \text{ with } j \in \mathcal{N}_i;\ \widehat{y}_i = y_i(t^i_k), \text{ for } t \in [t^i_k, t^i_{k+1}].$$

- The error between the actual value and held value can be written

$$e_i(t) = y_i(t) - \widehat{y}_i$$

- Output synchronization can be achieved if each agent transmits its current output y_i when the error grows large enough to meet the following triggering condition

$$\left\|e_i(t)\right\|_2 > \frac{\delta_1 \sum_{j \in \mathcal{N}_i} \left\|\widehat{y}_j - \widehat{y}_i\right\|_2^2}{\left\|\sum_{j \in \mathcal{N}_i} (\widehat{y}_j - \widehat{y}_i)\right\|_2},\ \forall t \geq 0$$

- Weak connectivity is still assumed

Event-driven Communication with Quantization

- When there is quantization in the network, the agents cannot synchronize exactly but the error can be bounded
- The same update law can be used

$$u_i(t) = \sum_{j \in \mathcal{N}_i} a\big[q(\widehat{y}_{k_j}) - q(\widehat{y}_{k_i})\big],$$

- Each agent transmits its current output when the following triggering condition is satisfied

$$\|e_i(t)\|_2 > \delta_3\big(\frac{1-\kappa}{2} - \frac{1}{2\beta}\big)\frac{1}{|\mathcal{N}_i|}\sum_{j \in \mathcal{N}_i} \big\|q(\widehat{y}_{k_j}) - q(\widehat{y}_{k_i})\big\|_2,$$

where $\delta_3 \in (0,1)$, $0 < \kappa < 1$ and $1 < \frac{1}{1-\kappa} < \beta$,

- It can be shown that the error in the output synchronization algorithm is bounded by the quantization error

Passivity and Dissipativity in Networked Switched Systems

Passivity for Switched Systems

- The notion of passivity has been defined for switched systems

$$\dot{x} = f_\sigma(x,u)$$
$$y = h_\sigma(x,u)$$

A *switched system* is *passive* if it meets the following conditions

1. Each subsystem *i* is passive when active:

$$\int_{t_1}^{t_2} u^T y dt \geq V_i(x(t_2)) - V_i(x(t_1))$$

2. Each subsystem *i* is dissipative w.r.t. ω_j^i when inactive:

$$\int_{t_1}^{t_2} \omega_j^i(u,y,x,t)dt \geq V_j(x(t_2)) - V_j(x(t_1))$$

3. There exists an input *u* so that the cross supply rates (ω_j^i) are integrable on the infinite time interval.

[McCourt & Antsaklis 2010 ACC, 2010 CDC]

Dissipativity for Switched Systems

Definition of Dissipativity in Discrete-time

A discrete-time switched system is dissipative if for each subsystem i there exists a positive function V_i such that the following conditions hold

1. Each subsystem i is dissipative while it is active with respect to $\omega_i(u, y)$:

$$\omega_i\big(u(t), y(t)\big) \geq V_i(x(t+1)) - V_i(x(t))$$

2. Each subsystem is dissipative when inactive with respect to $\omega_{ij}\ (u, y, x, t)$ for each active subsystem j:

$$\omega_{ij}\ (u(t), y(t), x(t), t(t)) \geq V_i(x(t+1)) - V_i(x(t))$$

Stability for Dissipative Discrete-time Systems

Theorem. *Dissipative switched systems are stable when $\omega_i < 0$ for all i and there exists an infinitely summable function $\phi(t)$ so that the inactive energy is bounded,*

$$\phi(t) \geq \omega_{ij}\ (u, y, x, t).$$

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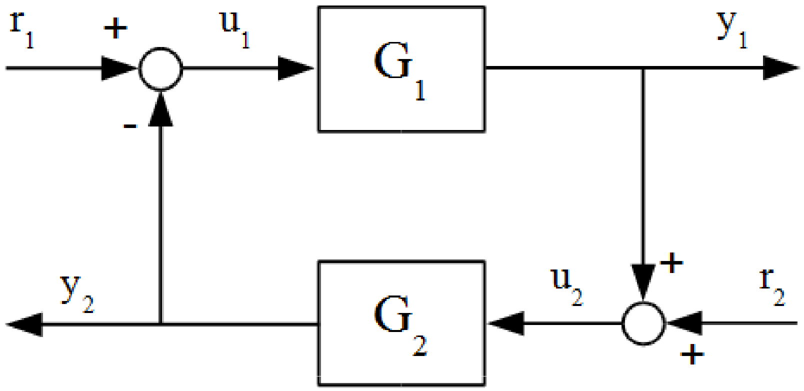
QSR Dissipativity for Switched Systems

- QSR dissipativity uses a quadratic supply rate to capture energy

$$\omega_i(u,y) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} Q_i & S_i \\ S_i^T & R_i \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix}$$

- Stability of switched systems can be assessed using Q_i



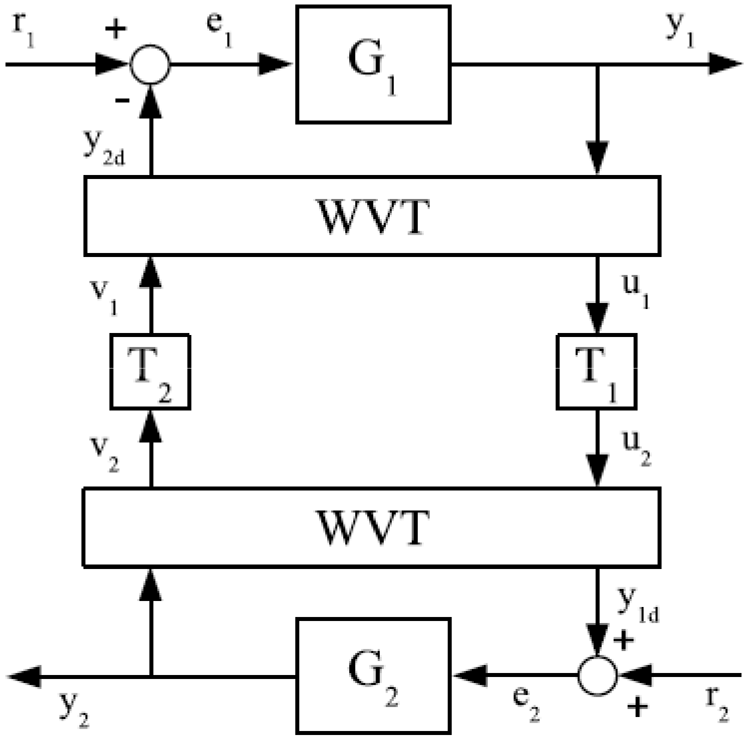
- Dissipativity of the feedback interconnection of two switched systems can be assessed with Q_i, S_i, R_i of both systems
- Large scale systems can be analyzed or designed using QSR dissipativity to ensure that the entire system is stable

- When dealing with passive switched systems ($Q_i = 0$, $S_i = {}^1\!/_2\, I$, $R_i = 0$), any sequential combination of systems in feedback or parallel can be shown to be passive and stable

[McCourt &Antsaklis 2012 ACC]

Stability of Networked Passive Systems

- When interconnecting passive discrete-time switched systems over a network, delays must be considered
- The transformation approach can be generalized to apply to switched systems
- The approach can compensate for time-varying delays
- The wave variable transformation is defined below



$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_1 \\ y_{2d} \end{bmatrix}$$

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_{1d} \\ y_2 \end{bmatrix}$$

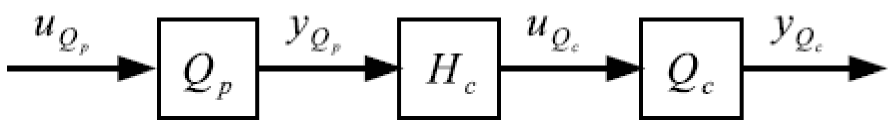
Compensating for quantization in passive systems

Motivating Problem

- When analyzing digital controllers or continuous systems that are sampled, discretization and quantization must be considered

- **There are existing results on how to preserve passivity when a continuous system is discretized**

- A passive system H_C may not be passive after input-output quantization

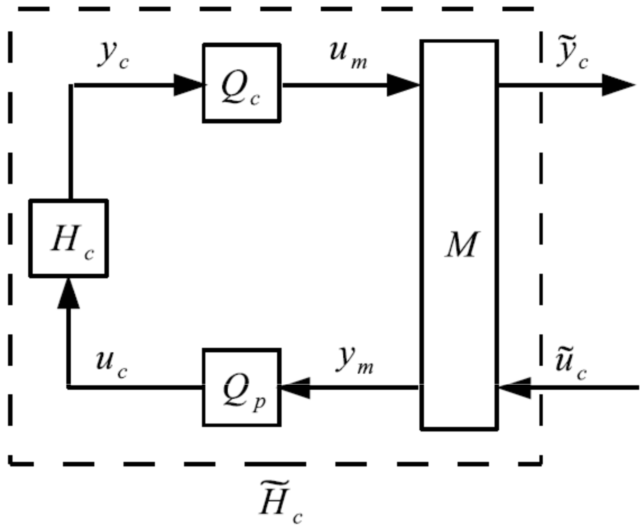


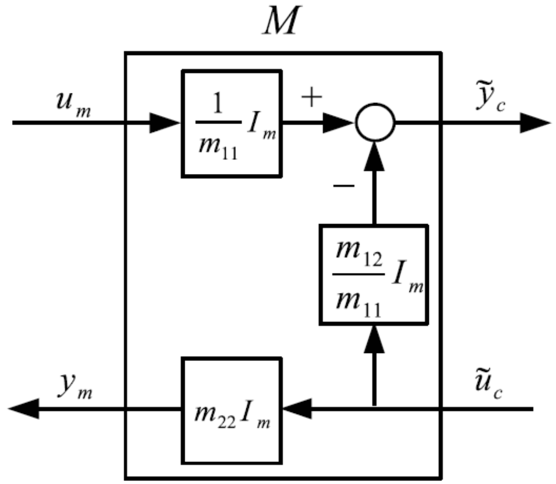
- Can recover passivity when making some simple assumptions on the quantization

One Solution to Quantization

- The given scheme recovers passivity for an output strictly passive (OSP) system (H_C) with passive quantizers (Q_C and Q_P) using an input/output transformation (M)

- Can be applied to passive switched systems
  - Switch input/output transformation according the current active system.
  - Stability conditions can be applied





[Zhu, Yu, McCourt,&Antsaklis 2012 HSCC]

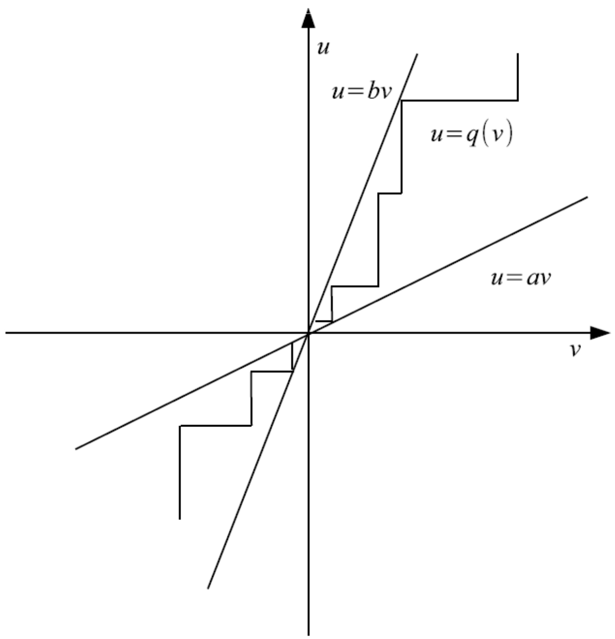
Class of Quantization

- This work applies with a restricted set of quantization – specifically quantizers that are restricted to lie in a cone

Definition *A quantizer is called a passive quantizer if its input v and output u satisfy*

$$av^2 \leq uv \leq bv^2$$

where $u = q(v)$ *and* $0 \leq a \leq b < \infty$.



- This definition requires very small values to map to zero

- Includes many practical quantizers

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Preserving Passivity…

THEOREM *Consider an OSP system H_C in the proposed scheme shown in Fig. 5 with passive quantizers Q_c and Q_p. If a transformation M is chosen such that*

$$m_{21} = 0, \quad m_{11}^2 = 2b_c^2$$

$$m_{11}m_{12} = \frac{-b_c^2}{\rho_c}, \quad m_{12}^2 = \frac{b_c^2 b_p^2}{\rho_c^2} m_{22}^2,$$

then the subsystem $\widetilde{H}_c : \tilde{u}_c \to \tilde{y}_c$ is output strictly passive such that

$$\Delta V_c(k) = V_c(k+1) - V_c(k) \leq \tilde{u}_c^T(k)\tilde{y}_c(k) - \rho_c \tilde{y}_c^T(k)\tilde{y}_c(k).$$

- This result is valid for continuous-time OSP systems.

- The negative feedback interconnection of two OSP systems is passive and thus stable.

-The same idea can be extended to switched systems

Extension to Switched Systems

THEOREM *Consider an output strictly passive discrete-time switched system H_C. This system is placed in the structure with passive quantizers defined by the constants a_c, b_c, a_p, and b_p. This control structure preserves the output strict passivity property of system H_C if the transformation $M(k)$ is chosen according to the following time-varying equations*

$$m_{21}(k) = 0, \quad m_{11}^2(k) = 2b_c^2$$

$$m_{11}m_{12}(k) = \frac{-b_c^2}{\rho(k)}, \quad m_{12}^2(k)(t) = \frac{b_c^2 b_p^2}{\rho^2(k)} m_{22}^2(k),$$

- This theorem guarantees that the switched system will be passive even with input and output quantization.

-Since the feedback interconnection of two passive switched systems is also passive and thus stable, this result can be used to further interconnect systems.

Models, Approximations and Passivity

- In the following passivity results on approximations that involve passivity indices

- **Modeling. Mathematical models and approximations.**

- How do we determine stability, and other properties of physical systems? Models and physical systems.

- Passivity in software. How do we define it so it is useful and makes sense.

Passivity and QSR-Dissipativity of a Nonlinear System and its ***Linearization***

Passivity and QSR-dissipativity Analysis of a System and its ***Approximation***

Problem Statement

- Motivation: *tradeoff* between model accuracy and tractability.
- Examples: linearization; feedback linearization; model reduction...
- Principle: *preserve* some fundamental properties or features: ***passivity***, stability, Hamiltonian structure...

- System Model:

$$u \rightarrow \boxed{\Sigma_1} \rightarrow y_1$$

$$u \rightarrow \boxed{\Sigma_2} \rightarrow y_2 = y_1 + \Delta y$$

  – view Σ_1 as the system we are interested in and view Σ_2 as an approximated model
  – the error is given through Δy (maybe modeling, linearization...)

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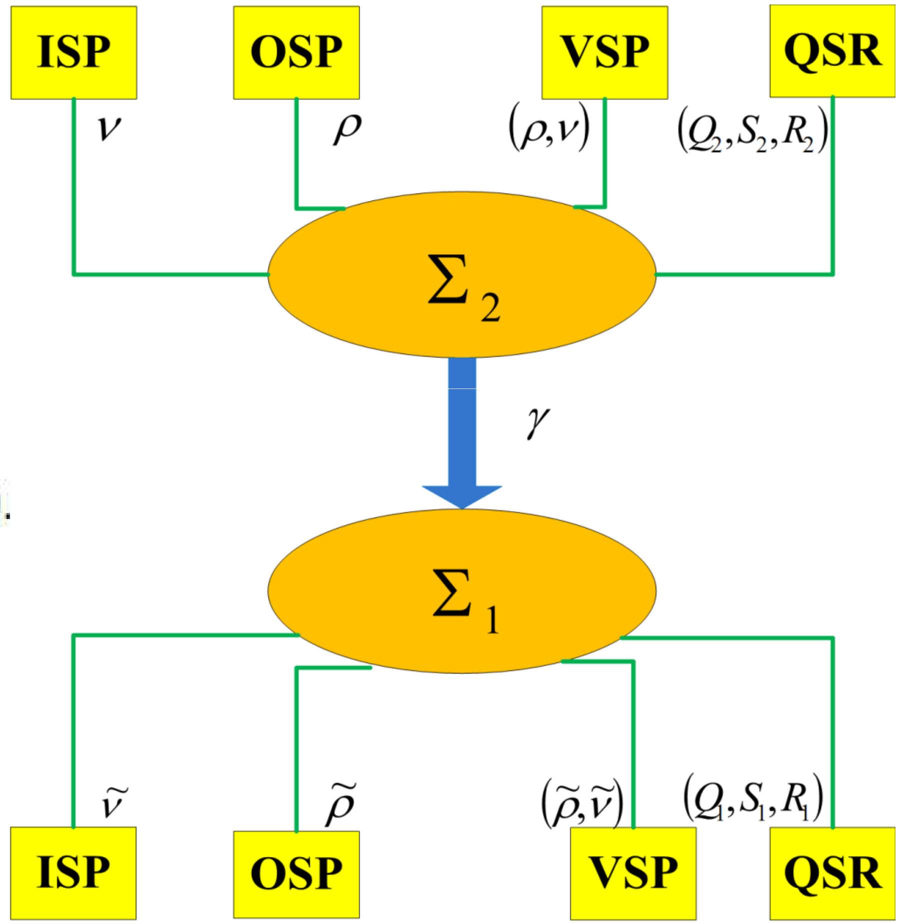
Problem Statement contd

- Suppose the approximate model is: ISP/OSP/VSP (having an ***excess*** of passivity) or QSR dissipative

- Suppose the error between the two systems is ***small*****,** i.e.

$$\|\Delta y\|_T \leq \gamma \|u\|_T, \quad \forall u \text{ and } \forall T \geq 0.$$

- ***The interested system***:
  - *Passive?*
  - *How passive?*
  - *QSR dissipative?*



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Main Results 1: ISP

- *Input strictly passive*:
  - ***passivity level:***

    Theorem 1 (ISP): Consider Σ_1 and Σ_2 in Fig. 1. Suppose (8) is satisfied for some $\gamma > 0$. If Σ_2 has IFP(ν) and $\gamma < \nu$, then, Σ_1 will be ISP for $\tilde{\nu} = \nu - \gamma$.

  - ***passive***:

    Corollary 1: Consider Σ_1 and Σ_2 in Fig. 1. Suppose (8) is satisfied for some $\gamma > 0$. If Σ_2 has IFP(ν) and $\gamma \leq \nu$, then, Σ_1 will be passive.

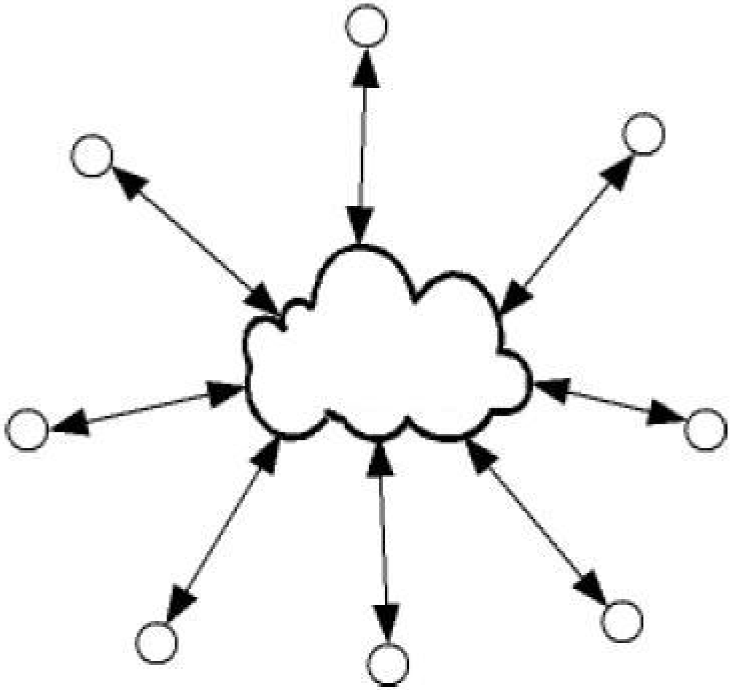
Symmetry in Systems

- Symmetry: A basic feature of shapes and graphs, indicating some degree of repetition or regularity
  - (Approximate) symmetry in characterizations of information structure
  - (Approximately) identical dynamics of subsystems
  - Invariance under group transformation e.g. rotational symmetry

- Why Symmetry?
  - Decompose into lower dimensional systems with better understanding of system properties such as stability and controllability
  - Construct symmetric large-scale systems without reducing performance if certain properties of low dimensional systems hold

Simple Examples

Star-shaped Symmetry and Hierarchies



$$u = u_e - \tilde{H} y$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

$$u_0 = u_{e0} - H y_0 - b y_1 - \cdots - b y_m$$

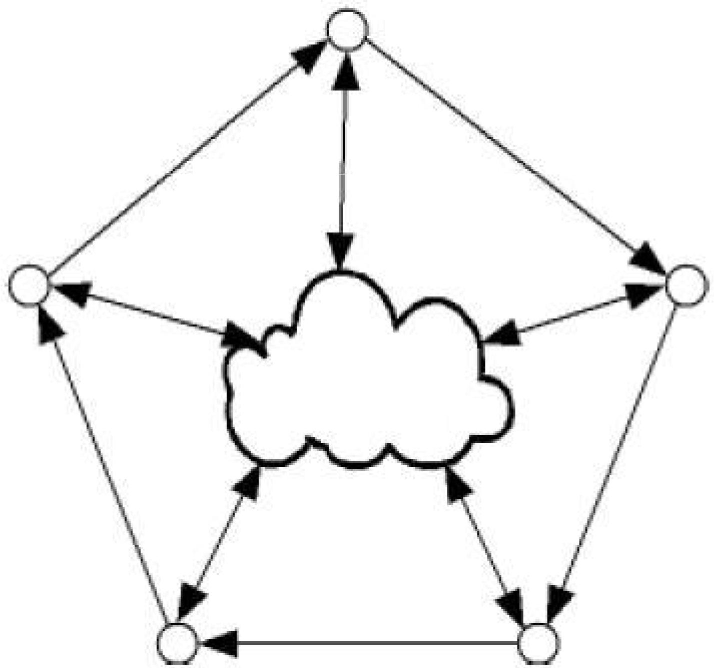
$$u_1 = u_{e1} - c y_0 - h y_1$$

$$\vdots$$

$$u_m = u_{em} - c y_0 - h y_m$$

Simple Examples

Cyclic Symmetry and Heterarchies



$$u = u_e - \tilde{H} y$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

$$\tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

$$u_0 = u_{e0} - H y_0 - b y_1 - \cdots - b y_m$$

$$u_1 = u_{e1} - c y_0 - v_1 y_1 - v_2 y_2 - \cdots - v_m y_m$$

$$\vdots$$

$$u_m = u_{em} - c y_0 - v_2 y_1 - v_3 y_2 - \cdots - v_1 y_m$$

Main Result (1)

Theorem (Star-shaped Symmetry)
Consider a (Q, S, R) – dissipative system extended by *m* star-shaped symmetric (q, s, r) – dissipative subsystems. The whole system is finite gain input-out stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(c^T rc + \beta(\hat{q} - b^T Rb)^{-1} \beta^T)}, \frac{\hat{q}}{b^T Rb}\right)$$

where

$$\hat{Q} = -H^T RH + SH + H^T S^T - Q > 0$$

$$\hat{q} = -h^T rh + sh + h^T s^T - q > 0$$

$$\beta = Sb + c^T s^T - H^T Rb - c^T rh$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

Main Result (2)

Theorem (Cyclic Symmetry)
Consider a (Q, S, R) – dissipative system extended by *m* cyclic symmetric (q, s, r) – dissipative subsystems. The whole system is finite gain input-out stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\bar{\sigma}(c^T rc + \beta_m \Lambda^{-1} \beta_m{}^T)}, \frac{-r\sigma(\tilde{h})\overline{\sigma(\tilde{h})} + s(\sigma(\tilde{h}) + \overline{\sigma(\tilde{h})}) - q}{b^T Rb}\right)$$

where

$$\tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

$$\sigma(\tilde{h}) = \sum_{j=1}^{m} v_j \lambda_i^j = \sum_{j=1}^{m} v_j e^{\frac{2\pi ij}{m}}$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

Main Result (3)

(cont.)

$$\Lambda = -r\tilde{h}^T\tilde{h} + s(\tilde{h}^T + \tilde{h}) - q \otimes I_m - b^T Rb \otimes circ([1,1,\ldots,1])$$

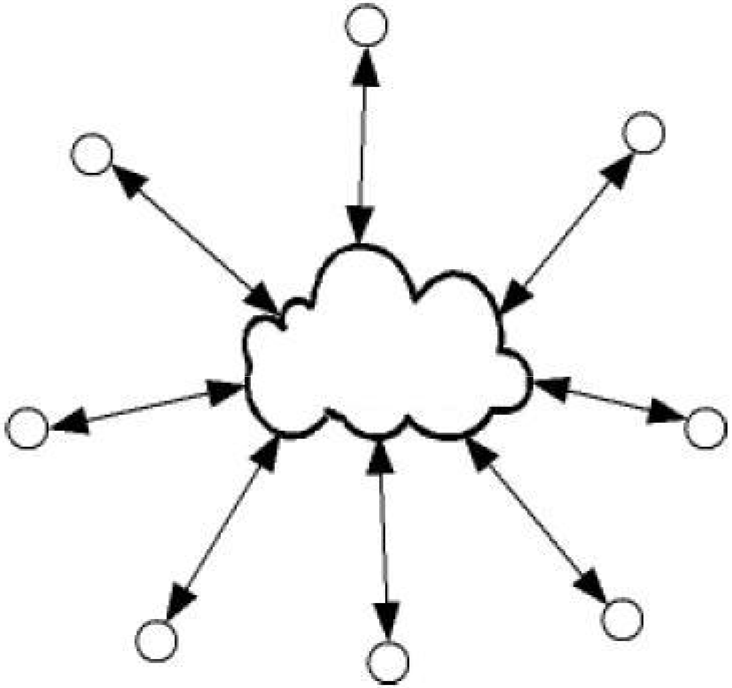
$$\beta = Sb + c^T s^T - H^T Rb - c^T r\tilde{h}$$

$$\beta_m = [\underbrace{\beta\beta\ldots\beta}_{m}]$$

the spectral characterization of $\tilde{h}$ should satisfy

$$\| \sigma(\tilde{h}) - \frac{s}{r} \| < \sqrt{\frac{s^2}{r^2} - \frac{q + mb^T Rb}{r}}$$

Simple Examples



$$u = u_e - \tilde{H}y$$

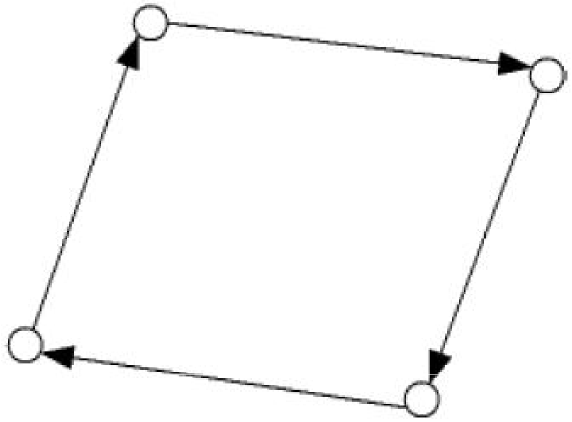
$$\tilde{H} = \underbrace{\begin{bmatrix} 0.9 & -0.8 & -0.8 & \cdots & -0.8 \\ -0.8 & 0.1 & 0 & \cdots & 0 \\ -0.8 & 0 & 0.1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -0.8 & 0 & 0 & \cdots & 0.1 \end{bmatrix}}_{m+1}$$

$$Q = q = -I, S = s = 0, R = r = \tfrac{1}{4} I$$

$$\Rightarrow \quad m < \min(3.11, 6.25) = 3.11$$

Remark: $(-I, 0, \alpha^2 I) -$ dissipative systems corresponding to systems with gain less or equal to α (here $\alpha = \tfrac{1}{2}$)

Simple Examples



$$u = u_e - \tilde{H}y \qquad q = 0, s = \tfrac{1}{2}, r = 1$$

$$\tilde{H} = \tilde{h} = \underbrace{\begin{bmatrix} 0.1 & 0.2 & 0 & \cdots & 0 \\ 0 & 0.1 & 0.2 & \cdots & 0 \\ 0 & 0 & 0.1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.2 & 0 & 0 & \cdots & 0.1 \end{bmatrix}}_{m}$$

The cyclic symmetric system is stable if

$$|\,\sigma(\tilde{h}) - \frac{s}{r}\,\| = \| \sum_{j=0}^{m-1} v_j e^{\frac{2\pi ij}{m}} \| \le 0.3 < 0.5 = \sqrt{\frac{s^2}{r^2} - \frac{q}{r}}$$

The above stability condition is always satisfied. Also

$$m < \min(+\infty, +\infty)$$

Thus the system can be extended with infinite numbers of subsystems without losing stability.

Main Result (4)

Theorem (Star-shaped Symmetry for Passive Systems)
Consider a passive system extended by *m* star-shaped symmetric passive subsystems. The whole system is finite gain input-output stable if

$$m < \frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(\beta \hat{Q}^{-1} \beta^T)}$$

where

$$\hat{Q} = \frac{H + H^T}{2} > 0$$

$$\hat{q} = \frac{h + h^T}{2} > 0$$

$$\beta = \frac{b + c^T}{2}$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

Main Result (5)

Theorem (Cyclic Symmetry for Passive Systems)
Consider a passive system extended by *m* cyclic symmetric passive subsystems. The whole system is finite gain input-output stable if

$$m < \frac{\underline{\sigma}(\hat{Q})}{\bar{\sigma}(\beta_m \Lambda^{-1} \beta_m^{\ T})}$$

where

$$\hat{Q} = \frac{H + H^T}{2} > 0 \qquad \Lambda = \frac{\tilde{h} + \tilde{h}^T}{2}$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

$$\beta = \frac{b + c^T}{2} \qquad \beta_m = [\underbrace{\beta\beta\ldots\beta}_{m}]$$

$$\sigma(\tilde{h}) = \sum_{j=1}^{m} v_j \lambda_i^j = \sum_{j=1}^{m} v_j e^{\frac{2\pi i j}{m}} \qquad \tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

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Concluding Remarks

- CPS, Distributed, Embedded, Networked Systems. Analog-digital, large scale, life cycles, safety critical, end to end high-confidence.
- Models, robustness, fragility, resilience, adaptation.
- New ways of thinking needed to deal effectively with the CPS problems. New ways to determine research directions.
- Passivity/Dissipativity and Symmetry are promising
- Circuit theory and port controlled Hamiltonian systems.
- Connections to Autonomy and Human in the Loop