

Classical Network Synthesis Revisited

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The talk consists of four parts. In the first part, the motivation is explained for revisiting certain questions in circuit theory, in particular, the synthesis theory of “one-port” (i.e. with a pair of external driving terminals) RLC networks and the questions of minimality associated with the Bott-Duffin procedure. The motivation relates to the synthesis of passive mechanical impedances and the need for a new ideal modelling element the “inertor” [6]. The development of the inertor from a mathematical concept and ideal modelling element through to its adoption as a standard component in Formula One racing and beyond is described [2].

In the second part, classical results from electrical circuit synthesis are reviewed including the procedures of Foster, Cauer, Brune, Darlington, Bott and Duffin. The reactance theorem of Foster [3] for lossless networks and the Bott-Duffin construction [1] for arbitrary positive-real functions are highlighted.

In the third part, the concept of regular positive-real functions is described. A positive-real function $Z(s)$ is defined to be *regular* if the smallest value of $\operatorname{Re}(Z(j\omega))$ or $\operatorname{Re}(Z^{-1}(j\omega))$ occurs at $\omega = 0$ or $\omega = \infty$. It is shown how the concept can aid the classification of low-complexity networks [4].

The fourth part describes recent work and new results on network synthesis [5].

REFERENCES

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