Self-Triggered Control Design with Guaranteed Performance

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TU/e Technische Universiteit Eindhoven University of Technology

Where innovation starts

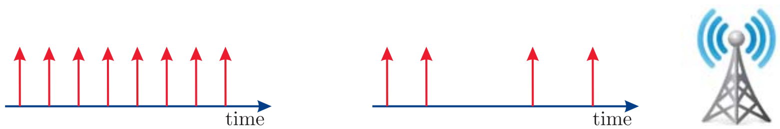
Acknowledgements

- Tom Gommans
- Duarte Antunes
- Paulo Tabuada
- Tijs Donkers

Introduction

Periodic or Aperiodic: That's the question!

- Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control



- Technological motivation:
  Resource-constrained large-scale cyber-physical systems
  - Computation time on embedded systems
  - Network utilisation in NCSs
  - Battery power in WCSs

Introduction

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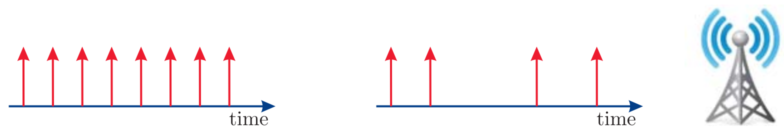

- Technological motivation:
  Resource-constrained large-scale cyber-physical systems
  - Computation time on embedded systems
  - Network utilisation in NCSs
  - Battery power in WCSs
- Fundamental motivation:
  - What is "optimal" sampling pattern for control purposes?

Introduction

Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

- Self-triggered control:

$$u(t) = \mathcal{U}(x(t_l)), \text{ when } t \in [t_l, t_{l+1})$$

$$t_{l+1} = t_l + \mathcal{M}(x(t_l))$$

Introduction

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Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

- Open issues:
  - Performance guarantees
    - Improving upon periodic control
  - Co-design

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Problem setup

- Linear system:

$$x_{t+1} = Ax_t + Bu_t$$

- Control costs:

$$J_{\text{cont}} = \sum_{t=0}^{\infty} x_t^\top Q x_t + u_t^\top R u_t$$

- Communication costs:

$$J_{\text{comm}} \sim f_{\text{ave}} = \frac{1}{h_{\text{ave}}}$$

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Two variants

(A) $\min J_{\text{comm}}$ s.t. $J_{\text{cont}} \leq c_{\text{cont}}$

(B) $\min J_{\text{cont}}$ s.t. $J_{\text{comm}} \leq c_{\text{comm}}$

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Two variants

(A) $\min J_{\text{comm}}$ s.t. $J_{\text{cont}} \leq c_{\text{cont}}$ – Self-triggered LQR [1]

(B) $\min J_{\text{cont}}$ s.t. $J_{\text{comm}} \leq c_{\text{comm}}$ – Roll-out LQR [2]

[1] Gommans, Heemels, Donkers, Tabuada, submitted
[2] Antunes, Heemels, Tabuada, CDC12

Self-triggered LQR

Standard LQR solution

$$x_{t+1} = Ax_t + Bu_t \qquad J_{\text{cont}} = \sum_{t=0}^{\infty} x_t^\top Q x_t + u_t^\top R u_t$$

- LQR Solution: $\min J_{\text{cont}}(x_0, \mathbf{u}) = V(x_0) = x_0^\top P x_0$
- P satisfying Discrete Algebraic Riccati Equation (DARE):

$$P = Q + A^\top PA - A^\top PB(R + B^\top PB)^{-1} B^\top PA$$

- Optimal controls given by feedback policy

$$u_t = K^* x_t \text{ with } K^* = (R + B^\top PB)^{-1} B^\top PA$$

Self-triggered LQR

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- LQR solution requires communication of states $x_t \in \mathbb{R}^{n_x}$ and updates of control actions $u_t \in \mathbb{R}^{n_u}$ at all times

Self-triggered LQR

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Objective: Given sub-optimality parameter $\beta \geq 1$, synthesize STC strategies that guarantee

$$J_{\text{cont}}(x_0, \mathbf{u}) = \sum_{t=0}^{\infty} x_t^\top Q x_t + u_t^\top R u_t \leq \underbrace{\beta V(x_0)}_{=c_{\text{cont}}}$$

and minimize communication ...

Self-triggered LQR

$$x_{t+1} = Ax_t + Bu_t$$

$$\begin{aligned} u(t) &= \bar{u}_l = \mathcal{U}(x(t_l)), \text{ when } t \in [t_l, t_{l+1}) \\ t_{l+1} &= t_l + \mathcal{M}(x(t_l)) \end{aligned}$$

How to synthesize $\mathcal{U}$ and $\mathcal{M}$ to guarantee

$$\sum_{t=0}^{\infty} x_t^\top Q x_t + u_t^\top R u_t \le \beta V(x_0)\,?$$

Sufficient: for each $l \in \mathbb{N}$

$$\sum_{t=t_l}^{t_{l+1}-1} x_t^\top Q x_t + u_t^\top R u_t \le \beta[V(x_{t_l}) - V(x_{t_{l+1}})]$$

Self-triggered LQR

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Greedy approach: At update time t_l, find maximal t_{l+1} s.t. diss. inequality still satisfiable

Implementation

Control algorithm: At update time t_l in state x_{t_l}

- find maximal M^* among $M \in \mathbb{N}$ such that $x_{t_l}^\top (P_M - \beta P) x_{t_l} \le 0$
- take $u_t = K_{M^*} x_{t_l}$ for $t \in [t_l, t_{l+1})$
- go to sleep until time $t_{l+1} = t_l + M^*$

$\longrightarrow$ Rather simple implementation!

Example self-triggered LQR

- Double integrator

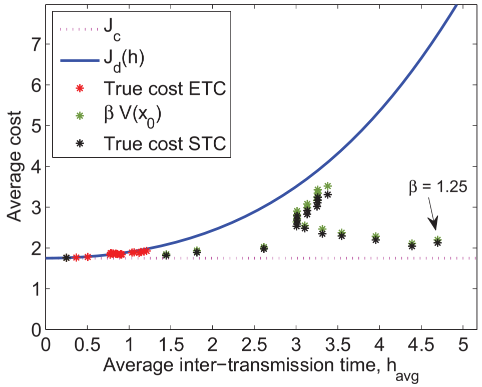
$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u = A_c x + B_c u$$

- Continuous-time costs

$$J_c(x_0, \mathbf{u}) = \int_0^\infty [x^\top Q_c x + u^\top R_c u] dt \text{ with } Q_c = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } R_c = 1$$

- Take $h = 0.25$ and discretize costs/plant exactly to obtain discrete-time costs/plant
- Various values for $\beta \ge 1$
- Compare to optimal discrete-time periodic LQR controller for different sampling period h

Example self-triggered LQR



- $\beta = 1.25$ minimal/average sampling time 1.25/4.70 with control costs 2.13
- periodic LQR with control costs ≈ 2.13 for $h = 1.53$

Roll-out LQR

- Linear system:

$$x_{t+1} = Ax_t + Bu_t$$

- Control costs:

$$J_{\text{cont}} = \sum_{t=0}^{\infty} x_t^\top Q x_t + u_t^\top R u_t$$

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[1] D. Antunes, W. P. M. H. Heemels, P. Tabuada, CDC12

Roll-out LQR

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- Communication costs: $J_{\text{comm}} \sim f_{\text{ave}} = \frac{1}{h_{\text{ave}}}$

$$x_{t+1} = \begin{cases} Ax_t + Bu_t, & \text{when } u_t \text{ transmitted at time } t\ (\sigma_t = 1) \\ Ax_t + Bu_{t-1}, & \text{when } u_t \text{ not transmitted at time } t\ (\sigma_t = 0) \end{cases}$$

Problem: Design control/scheduling policy $\pi = \{(\mu_t^\sigma, \mu_t^u)\}_{t\in\mathbb{N}}$ with

$$(\sigma_t, u_t) = (\mu_t^\sigma(x_t), \mu_t^u(x_t))$$

minimizing J_{cont} s.t. $J_{\text{comm}} \leq c_{\text{comm}} = \frac{1}{q}$ (i.e., $h_{\text{ave}} \geq q$)

Roll-out LQR: The main idea

- Receding horizon: Scheduling times $t_k = kh$ with h sched. period
- Optimal control problem: based on state $x_{t_k} = x$ minimize J_{cont} over admissible schedules $\{\sigma_t^j\}_{t\in\mathbb{N}}$, $j = 1, 2, \ldots, J$, and inputs $\{u_t\}_{t\in\mathbb{N}}$

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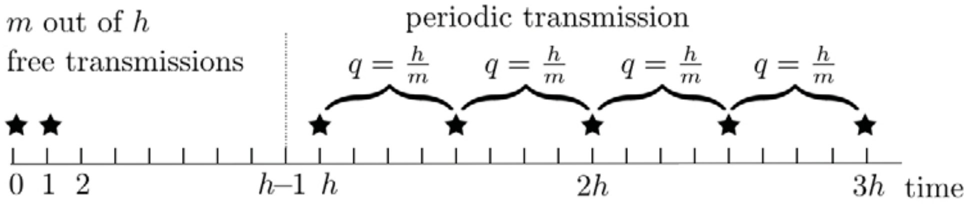
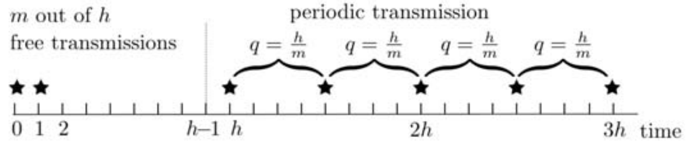
  - Free choice: $m = \frac{h}{q}$ transmissions at $0, 1, 2, \dots, h-1$
  - Roll-out algorithm: after time h periodic transmission with period $q = \frac{h}{m}$



Roll-out LQR



- Resulting scheduling/control policy: At scheduling time $t_k = kh$ and state $x_{t_k} = x$

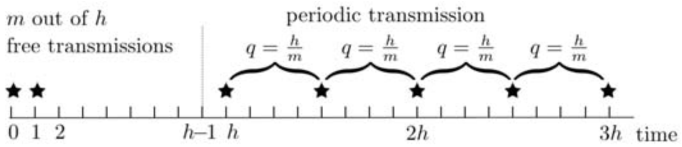
$$j^* = \text{arg min}\{x^\top P_j x \mid j = 1, 2, \dots, J\}$$

with $x^\top P_j x$ the optimal costs corresponding to schedule $\{\sigma_t^j\}_{t\in\mathbb{N}}$

$$\sigma_t = \sigma_t^{j^*}$$

$$u_t = K_{t,j^*} x_t, \; t \in \mathbb{N}_{[kh,(k+1)h)}$$

Roll-out LQR



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- This policy results in $h_{ave} = q$ and thus $J_{\text{comm}} = \frac{1}{q}$
- Outperforms periodic schedule with period q!

Roll-out LQR: Numerical example

$$\dot{x} = A_C x + B_C u$$

with

$$A_C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\kappa_m & \kappa_m & 0 & 0 \\ \kappa_m & -\kappa_m & 0 & 0 \end{bmatrix}, \quad B_C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \; \kappa_m = 2\pi^2$$

- Control costs: $\int_0^\infty (x_1^2 + x_2^2 + 0.1 u_C^2) dt$

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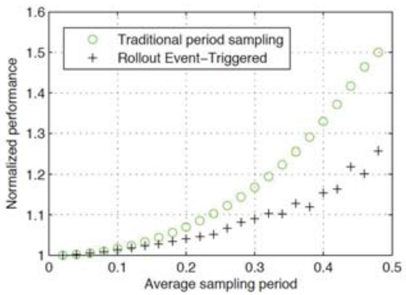

Conclusions

- Self-triggered LQR [1] $\min J_{\text{comm}}$ s.t. $J_{\text{cont}} \leq c_{\text{cont}}$
  - Dissipation inequality guaranteeing suboptimal performance
  - "PWL control law" $u = K_M x$ for largest M s.t. $x^\top (P_M - \beta P) x \leq 0$

[1] Gommans, Heemels, Donkers, Tabuada, submitted
[2] Antunes, Heemels, Tabuada, CDC12

Conclusions

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  - Dissipation inequality guaranteeing suboptimal performance
  - "PWL control law" $u = K_M x$ for largest M s.t. $x^\top (P_M - \beta P) x \leq 0$
- Roll-out LQR [2] $\min J_{\text{cont}}$ s.t. $J_{\text{comm}} \leq c_{\text{comm}}$
  - Receding horizon control law based on roll-out
  - Guaranteed to outperform periodic base policy
  - "PWL control/scheduling structure"

$$j^* = \text{arg min}\{x^\top P_j x \mid j = 1, 2, \ldots, J\}$$

$$u_t = K_{t,j^*} x_t, \text{ and } \sigma_t = \sigma_t^{j^*} \text{ for } t \in \mathbb{N}_{[kh,(k+1)h)}$$

- Simulations look very promising for both!

[1] Gommans, Heemels, Donkers, Tabuada, submitted
[2] Antunes, Heemels, Tabuada, CDC12

Outlook & Extensions

- Including constraints: Self-triggered MPC [3]
- Event-triggered LQR [1]
- Roll-out LQR: Including stochastic disturbances [2]
- Roll-out LQR: Guaranteeing strict improvement w.r.t. periodic
- Robustness

[1] Gommans, Heemels, Donkers, Tabuada, submitted
[2] Antunes, Heemels, Tabuada, CDC12
[3] Barradas-Berglind, Gommans, Heemels, NMPC12

Suggested reading

Related to this talk ...

- T.M.P. Gommans, W.P.M.H. Heemels, M.C.F. Donkers, P. Tabuada, *Event-Triggered and Self-Triggered LQR Control*, submitted
- D. Antunes, W.P.M.H. Heemels, P. Tabuada, *Dynamic Programming Formulation of Periodic Event-Triggered Control: Performance Guarantees and Co-Design*, IEEE Conference on Decision and Control 2012, Hawaii, USA.
- J.D.J. Barradas Berglind, T.M.P. Gommans, W.P.M.H. Heemels, "Self-triggered MPC for constrained linear systems and quadratic costs," IFAC Conference on Nonlinear Model Predictive Control 2012, Noordwijkerhout, Netherlands (invited semi-plenary lecture), p. 342-348.

... and beyond ...

- W.P.M.H. Heemels, M.C.F. Donkers, and A.R. Teel, *Periodic Event-Triggered Control for Linear Systems*, IEEE Transactions on Automatic Control , April 2013, to appear.
- W.P.M.H. Heemels and M.C.F. Donkers, *Model-Based Periodic Event-Triggered Control for Linear Systems*, Automatica, accepted.
- M.C.F. Donkers and W.P.M.H. Heemels, *Output-Based Event-Triggered Control with Guaranteed L_∞-gain and Improved and Decentralised Event-Triggering*, IEEE Transactions on Automatic Control, 57(6), p. 1362-1376.
- W.P.M.H. Heemels, J.H. Sandee, P.P.J. van den Bosch, *Analysis of event-driven controllers for linear systems*, International Journal of Control, 81(4), pp. 571-590 (2008).
- W.P.M.H. Heemels, R.J.A. Gorter, A. van Zijl, P.P.J. v.d. Bosch, S. Weiland, W.H.A. Hendrix, M.R. Vonder, *Asynchronous measurement and control: a case study on motor synchronisation*, Control Engineering Practice, 7(12), 1467-1482, (1999).

Preprints of papers downloadable from:

`http://www.dct.tue.nl/heemels`