# Theoretical and Experimental Investigation of Stratified Robotic Finger Gaiting and Manipulation*

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**Abstract**

This paper presents the application of stratified motion planning to the robotic manipulation problem. Although the manipulation problem is a subclass of applications for stratified motion planning, the method present is general in that it is formulation in a manner independent of the object surface geometry or the kinematics of the "fingers" of the robot. The theoretical development of the method is presented as well as experimental results.

## 1 Introduction

This paper presents the development and experimental verification of a general control framework for robotic grasping and manipulation problems where "fingers" manipulate a grasped object. The method incorporates standard techniques from nonlinear control; furthermore, it *analytically* incorporates techniques to exploit the discontinuities present if the fingers intermittently contact the object (such manipulation has been called "finger gaiting"). Incorporating the discontinuities of the equations of motion of a system into a general motion planning algorithm is difficult because almost all motion planning methods assume that the equations of motion are smooth.

Robotic grasping and manipulation have been the subject of many research efforts, and only an overview can be provided here. Vast efforts have been directed toward the *analysis* of grasp stability and force closure [28, 29, 32], motion planning assuming continuous contact [22, 37, 13] and haptic interfaces and other sensing [4, 31, 30]. *Finger gaiting*, where fingers make and break contact with the object has been less extensively considered and is the main focus of this paper. Finger gaiting has been implemented in certain instances [26, 15, 5] and also partially considered theoretically [14, 3, 10]. Not related to finger gaiting, however, perhaps the approach which most closely mirrors that of the subject of this paper is in [27] where notions of controllability and observability from "standard" control theory are applied to grasping. Additionally closely related is the work in [13] where the fundamental grasping constraint from [23] is slightly modified to include controlled relative velocities.

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This presentation of background material assumes that the reader is familiar with concepts from grasping as well as nonlinear control from [16, 23].

## 2.1 Rolling Contact Kinematics

Most grasping motion planning or manipulation algorithms (such as [25]) or analysis (such as [27]) techniques which allow the fingers to roll relative to the object surface have been formulated in *contact coordinates*. In particular, the differential equations relating the evolution of contact to the relative velocities between a finger tip and object well known and are given by [22, 23]:

$$\begin{aligned} \dot{\alpha}_f &= M_f^{-1}\left(K_f+\tilde{K}_o\right)^{-1}\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} \\ \dot{\alpha}_o &= M_f^{-1}R_\psi\left(K_f+\tilde{K}_o\right)^{-1}\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} \\ \dot{\psi} &= \omega_z + T_f M_f \dot{\alpha}_f + T_o M_o \dot{\alpha}_o, \end{aligned} \tag{1}$$

where $\alpha_f = (u_f, v_f)$, $\alpha_o = (u_o, v_o) \in \mathbb{R}^2$ are the contact coordinates which parameterize the finger and object, respectively, the $M$'s, $T$'s and $K$'s are the metric, torsion, and curvature forms, respectively, describing the geometry of the object or finger surfaces, denoted by subscript $o$ or $f$, respectively, and the $\omega$'s are components of the relative angular velocities between frames affixed to the body and object at the point of contact.

The kinematic constraints which relate the relative velocities of the finger and object to joint velocities, which, coupled with Equation 1 provide a complete description of the manipulation dynamics. The coordinate frames used to describe the grasping manipulation are the standard frames from [23], and include the *palm frame*, $P$, a *station frame*, $S_i$, associated with each finger, a *finger frame*, $F_i$, associated with each finger and an *object frame*, $O$, used to describe the configuration of the object. Also, defined at every point on the surface of the object and finger tips are a family of Gauss frames, denoted by $L_o$ and $L_{f_i}$. These frames are *fixed* with respect to the object and fingers, respectively. Also, define at the point of contact the Gauss frames $C_o$ and $C_{f_i}$ attached to the object and finger tips respectively which move with the point of contact.

Assuming that contact friction is sufficient to prevent slipping, then the directions in which forces can be applied are exactly the same components in which the relative velocity between $L_o$ and $L_{f_i}$ must be zero at the $i$th contact point, *i.e.*, $B^T V^b_{l_o l_f} = 0$, where $V^b_{l_o l_f}$ is the body velocity of frame $L_f$ with respect to $L_o$ and $B$ is the *wrench basis* (see [23] for a complete explanation).

We consider a modified system where the wrench basis is appended with two additional columns that encode the fact that the relative rolling velocities (from Equation 1) are constrained to be a specified value. In particular, for point contact with friction,

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

angular velocity between the $L$ frames on the object and finger. Note that for this contact model, only $\omega_z$ is unconstrained.

For the case where the relative angular velocities are going to be specified,

$$B^T V^b_{l_o l_f} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_x \\ \omega_y \end{bmatrix} = \xi, \tag{2}$$

where $\omega_x$ and $\omega_y$ are determined (as illustrated subsequently) from Equation 1. A more useful expression will result from rewriting Equation 2 in terms of the velocity of the object and the joint velocity of the fingers, and a simple derivation results in

$$-B^T \mathrm{Ad}^{-1}_{g_{pl_f}} \mathrm{Ad}_{g_{po}} V^b_{po} + B^T \mathrm{Ad}^{-1}_{f_i l_f} J^b_{s_i f_i} \dot{\theta}_i = \xi, \tag{3}$$

where Ad is the adjoint transformation, $V^b_{ab}$ is the body velocity of frame $b$ relative to frame $a$ and $J^b_{ab}$ is the body Jacobian. Each is fully explained in [23].

## 2.2 Stratified Systems

One of the authors has previously considered nonholonomic motion planning and control for so-called stratified systems, which are systems that can switch among multiple contact states [11, 7, 8, 10, 6, 12]. Examples of stratified systems include the manipulation problem considered in this paper, legged locomotion and certain types of hybrid systems.

A simple example will provide an intuitive understanding of the geometry inherent in stratified systems. Consider the simplistic example two fingers intermittently engaging an object. The set of configurations corresponding to one of the robots engaging the object is a codimension one submanifold contained in the configuration space. The same is true when the other robot engages the object. Similarly, when both robots engage the object, the system is on a codimension two submanifold of the configuration space formed by the intersection of the single contact submanifolds. Each submanifold is referred to as a *stratum*. The structure of the configuration manifold for such a system is abstractly illustrated in Figure 1. Note that the equations of motion for the system will be *different* on each submanifold because the constraints on the system will be different on each submanifold.

By considering systems more general than the two cooperating robots in the example, a general definition of stratified configuration spaces can be developed. Let $S_0$ denote the system's entire configuration manifold and $S_i \subset S_0$ denote the codimension one submanifold of $S_0$ that corresponds to all configurations where only the $i$th robot engages the object. Denote, the intersection of $S_i$ and $S_j$, by $S_{ij} = S_i \cap S_j$. The set $S_{ij}$ physically corresponds to states where both the $i$th and $j$th robots engage the object. Further intersections can be similarly defined in a recursive fashion: $S_{ijk} = S_i \cap S_j \cap S_k = S_i \cap S_{jk}$, etc. The lowest-dimensional stratum (corresponding to all fingers in contact with the object for grasping problems) will be called the *bottom stratum*.

Definition 2.1: (Stratified configuration manifold)

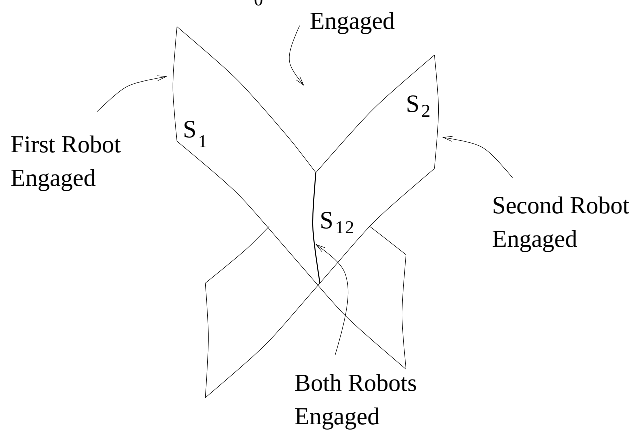


**Figure 1.** Configuration manifold structure for two cooperating robots.

Let $S_0$ be a manifold, and $n$ functions $\Phi_i : S_0 \mapsto \mathbb{R}$, $i = 1, \ldots n$ be such that the level sets $S_i = \Phi_i^{-1}(0) \subset S_0$ are regular submanifolds of $S_0$, for each $i$, and the intersection of any number of the level sets, $S_{i_1 i_2 \cdots i_m} = \Phi_{i_1}^{-1}(0) \cap \Phi_{i_2}^{-1}(0) \cap \cdots \cap \Phi_{i_m}^{-1}(0)$, $m \leq n$, is also a regular submanifold of $S_o$. Then $S_0$ and the functions $\Phi_n$ define a *stratified configuration space.*

## 2.3 Stratified Motion Planning

For smooth nonlinear systems, there are various motion planning techniques (piecewise constant inputs [17], steering with sinusoids [25, 24, 23, 36, 35], small amplitude inputs for mechanical system on Lie groups [19, 18, 2, 1], pushing, [20, 21], and others, [34, 33]). One of the authors has extended the method using piecewise constant inputs from [17] to the stratified case [12, 7] with application to legged robotic locomotion.

The basic approach is to consider the set of vector fields defined on the lowest-dimensional stratum (all fingers in contact) and to incorporate vector fields defined on higher strata by appropriately "projecting" them onto the bottom stratum. Then a series expansion (the *Chen–Flies* series) and the notion of the "extended system" (described subsequently) can be used in a straight–forward manner to construct control inputs which will steer the system to the final position with reference to a nominal trajectory.

For example, consider the simple cooperating robot configuration space as shown in Figure 1. Assume that on stratum $S_{12}$, (corresponding to both fingers in contact with the object) the vector field $g_{1,1}$ moves the system off of $S_{12}$ and onto $S_1$, (finger 2 disengages the object) and correspondingly, $g_{2,1}$ moves the system off of $S_{12}$ onto $S_2$ (finger 1 disengages the object). Also, consider the vector fields $g_{1,2}$ and $g_{2,2}$, defined on $S_1$ and $S_2$ respectively (corresponding to some motion of the system with fingers 2 and 1 not in contact with the object, respectively). Consider the following sequence of flows, starting from the point $x_0 \in S_{12}$

$$x_f = \underbrace{\phi_{-g_{2,1}}^{t_6}}_{S_{12} \leftarrow S_2} \circ \underbrace{\phi_{g_{2,2}}^{t_5}}_{\text{on } S_2} \circ \underbrace{\phi_{g_{2,1}}^{t_4}}_{S_2 \leftarrow S_{12}} \circ \underbrace{\phi_{-g_{1,1}}^{t_3}}_{S_{12} \leftarrow S_1} \circ \underbrace{\phi_{g_{1,2}}^{t_2}}_{\text{on } S_1} \circ \underbrace{\phi_{g_{1,1}}^{t_1}}_{S_1 \leftarrow S_{12}} (x_0), \tag{4}$$

as illustrated in Figure 2. The notation under each flow indicates what the flow is doing, *e.g.*, "$S_{12} \leftarrow S_1$" means that the flow takes the system from $S_1$ to $S_{12}$ and "on $S_1$" means

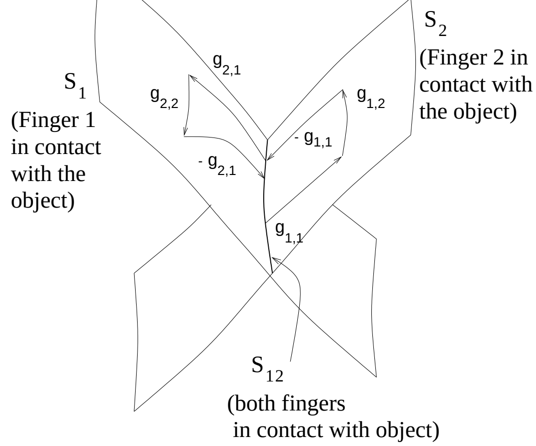


**Figure 2.** Sequence of flows.

that the flow was entirely on $S_1$. In this sequence of flows, the system first moved off of the bottom stratum into $S_1$, flowed along the vector field $g_{1,2}$, flowed back onto the bottom stratum, off of the bottom stratum onto $S_2$, along vector field $g_{2,2}$ and back to the bottom stratum. In robotic finger gaiting, such a sequence of flows corresponds to the following sequence of motions:

1. finger 2 disengaging the object;
2. some motion of the system with finger 1 in contact with the object and finger 2 not in contact with the object;
3. finger 2 engaging the object;
4. finger 1 disengaging the object;
5. some motion of the system with finger 2 in contact with the object and finger 1 not in contact with the object; and,
6. finger 1 engaging the object.

It is a basic result of differential geometry (the Campbell–Baker–Hausdorff formula), that if the Lie bracket between two vector fields is zero, then their flows commute. Thus, if

$$[g_{1,1}, g_{1,2}] = 0 \quad \text{and} \quad [g_{2,1}, g_{2,2}] = 0, \tag{5}$$

it is possible to reorder the above sequence of flows, by interchanging the flow along $g_{1,1}$ and $g_{1,2}$ and the flows along $g_{2,1}$ and $g_{2,2}$ as follows

$$x_f = \underbrace{\phi^{t_5}_{g_{2,2}} \circ \phi^{t_6}_{-g_{2,1}}}_{\text{interchanged}} \circ \phi^{t_4}_{g_{2,1}} \circ \underbrace{\phi^{t_2}_{g_{1,2}} \circ \phi^{t_3}_{-g_{1,1}}}_{\text{interchanged}} \circ \phi^{t_1}_{g_{1,1}}(x_0). \tag{6}$$

If $t_1 = t_3$ and $t_4 = t_6$, this reduces to

$$x_f = \underbrace{\phi^{t_4}_{g_{2,2}} \circ \phi^{t_2}_{g_{1,2}}}_{\text{on } S_{12}}(x_0). \tag{7}$$

Note that that $g_{1,2}$ and $g_{2,2}$ are vector fields in the equations of motion for the system on $S_1$ and $S_2$ respectively, (where each one of the fingers is not in contact), but *are not part of the equations of motion* on $S_{12}$ when both fingers are in contact, but, for motion in the same net displacement as Equation 4, where the system switched between strata.

In the above example, the vector fields that took the system off of a substratum correspond in the grasping case to moving a finger out of contact (or back into contact) with the object. Due to the fact that the fingers are assumed to be holonomic, the Lie bracket decoupling expressed in Equation 5 will *always* be satisfied.

# 3 Underactuated Manipulation

The equations of motion for the grasping system are of the form

$$\begin{aligned} -B^T \mathrm{Ad}^{-1}_{g_{pl_f}} \mathrm{Ad}_{g_{po}} V^b_{po} + B^T \mathrm{Ad}^{-1}_{f_i l_f} J^b_{s_i f_i} \dot{\theta}_i &= \xi \\ M_f^{-1} \left( K_f + \tilde{K}_o \right)^{-1} \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} &= \dot{\alpha}_f \\ M_f^{-1} R_\psi \left( K_f + \tilde{K}_o \right)^{-1} \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} &= \dot{\alpha}_o, \end{aligned} \tag{8}$$

for each finger in contact with the object. For fingers out of contact with the object, there is no constraint from the object, and the contact coordinates evolve according to

$$\begin{aligned} \dot{\alpha}_f &= M_f^{-1} \left( K_f + \tilde{K}_o \right)^{-1} \left( \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} - \tilde{K}_o \begin{bmatrix} v_x \\ v_y \end{bmatrix} \right) \\ \dot{\alpha}_o &= M_0^{-1} R_\psi \left( K_f + \tilde{K}_o \right)^{-1} \left( \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} + K_f \begin{bmatrix} v_x \\ v_y \end{bmatrix} \right) \\ \dot{\psi} &= \omega_z + T_f M_f \dot{\alpha}_f + T_o M_o \dot{\alpha}_o, \end{aligned} \tag{9}$$

for each finger out of contact with the object and with $V^b_{po} = 0$. The states that we desire to control are the object velocity, $V^b_{po}$ and the contact coordinates, $\alpha_f$ and $\alpha_o$ for each finger contact. Algebraically solving the above equations to separate them into an equation of the standard nonlinear control form:

$$\dot{x} = g_1(x) u_1 + \cdots + g_n(x) u_n, \tag{10}$$

and a set of state equations is theoretically straight-forward, but may be nearly intractable for complicated surface geometries.

A complete description of the motion planning algorithm from [17] and its extension to the stratified case in [7, 12, 9] is beyond the scope of this paper, so only a outline is provided here. Recall that the Lie bracket between two vector fields can be expressed by

$$[g_1, g_2](x) = \frac{\partial g_2(x)}{\partial x} g_1(x) - \frac{\partial g_1(x)}{\partial x} g_2(x),$$

and that the flow along the vector field corresponding to a Lie bracket motion can be approximated by

$$\phi^t_{[g_1,g_2]}(x_0) \approx \phi^{\sqrt{t}}_{-g_2} \circ \phi^{\sqrt{t}}_{-g_1} \circ \phi^{\sqrt{t}}_{g_2} \circ \phi^{g_1}_{\sqrt{t}}(x_0), \tag{11}$$

where $\phi^t_g$ represents the flow of the system along the vector field $g$ for time $t$, *i.e.*, the control corresponding to vector field $g$ is turned on for time $t$. Although the construction and formalism is substantial, the basic idea in [17] is to decompose a desired motion (called the nominal trajectory) into multiple subtrajectories along various vector fields these elements will be Lie brackets, and flowing in Lie bracket directions will need to be approximated in a manner expressed in Equation 11.

In particular, related to the original system (Equations 8 and 9) is *formal differential extended system*

$$\dot{S}(t) = S(t)\left(B_1 v_1 + \cdots + B_p v_p\right) \tag{12}$$

where the $B_i$'s belong to a noncommutative formal Lie algebra and are related to the original vector fields in Equation 10 and their Lie brackets. The extended system is picked so that the vector fields corresponding to the $B_i$'s are full rank at every point along the desired trajectory. Therefore, if the original system is underactuated, the extended system will contain Lie brackets directions along which the system cannot directly flow. Additionally, all flows of the original system can be represented (formally) by

$$S(t) = e^{h_p(t)B_p} e^{h_{p-1}(t)B_{p-1}} \cdots e^{h_1(t)B_1}, \tag{13}$$

where the $h_i$ are called the *backward Philip Hall coordinates* and, because of the formal representation, the exponentials can be expanded in the "standard" series expansion for exponentials. Differentiating Equation 13 with respect to time and equating the resulting coefficients of the $B_i$'s with the coefficients of the $B_i$'s in Equation 12 yields differential equations for that can be solved to determine the backward Philip Hall coordinates. Once the Philip Hall coordinates are computed, it is straight-forward to construct piecewise constant control inputs for the *original system* (Equation 9) to approximate the total flow of the system along $\gamma$, as was illustrated by Equation 11.. The method works exactly for *nilpotent systems* (nilpotency is a property of the Lie algebra containing the $g_i$). For the general, non-nilpotent case, the method works approximately and [17] derives explicit bounds on the resulting error.

Now, in the stratified grasping case, since the Lie bracket decoupling expressed in Equation 5 is always satisfied, motions when each of the fingers are out of contact with the object can be considered as part of the collection of vector fields that can be used for motion planning. Accordingly, we can define the *extended stratified system.*

Definition 3.1: (Extended Stratified System)

The *extended stratified system* on the bottom strata, $S_B$, is the driftless system comprised of the vector fields on the bottom strata, chosen vector fields from the higher strata, and Lie brackets of vector fields from $S_B$ and higher strata, *i.e.*, it is a system taking the form:

$$\dot{x} = b_1(x)v_1 + \cdots b_m(x)v_m + \underbrace{b_{m+1}v_{m+1} \cdots + b_n v_n}_{\text{from higher strata}} + \underbrace{b_{n+1}v_{n+1} + \cdots + b_p v_p}_{\text{any Lie brackets}}, \tag{14}$$

where the $\{b_1, \ldots, b_p\}$ span $T_x S_0$, the inputs $v_1, \ldots, v_n$ are real, and the inputs $v_{n+1}, \ldots, v_p$ are fictitious.

Specifically, the algorithm to generate trajectories that move the system from initial configuration $p$ to final configuration $q$ is as follows.

1. Construct the extended stratified system, Equation (14), on the bottom strata, $S_B$.

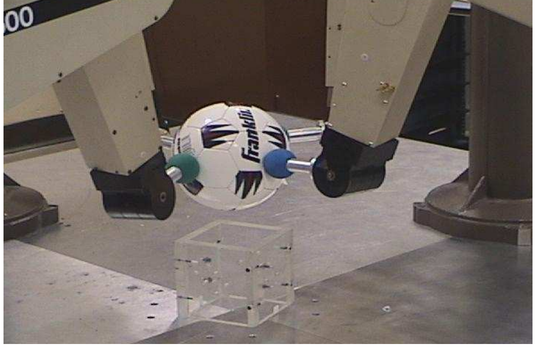

**Figure 3.** Experimental stratified manipulation.

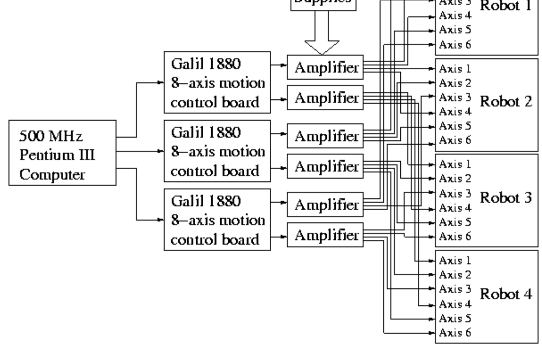


**Figure 4.** Schematic of control system.

2. Find a nominal trajectory, $\gamma(t)$, that connects $p$ and $q$. Given $\gamma(t)$, solve

$$\dot{\gamma}(t) = b_1(x)v_1 + \cdots + b_p(x)v_p,$$

for the fictitious inputs, $v_i$.

3. Solve the stratified extended system for the fictitious control inputs, *i.e.*, solve for the backward Philip Hall coordinates by solving the differential equations derived from from Equations 12 and 13.

4. For each path segment in each strata, compute the actual controls that steer the system along $\gamma(t)$.

5. Flow along each first order vector field, and approximate higher order vector fields as illustrated in Equation 11. In general, it will be necessary to switch strata between some of these flows.

# 4 Experimental Validation

The above results have also been verified and demonstrated experimentally. The experimental platform consists of four standard Puma 560 robots mounted on a common platform. All of the robots are controlled by a central 500 MHz Pentium III computer via Galil 1880 8-axis motion control boards operating six amplifies, each controlling four axes each. A close-up of the four robots with spherical finger tips engaging a spherical football is illustrated in Figure 3 and the complete system is schematically illustrated in Figure 4. Experiments are carried out with spherical and "egg-shaped" objects as spherical and flat finger tips with only four of the robot axes actuated. Any arbitrary axis of rotation can be specified and the ensuing motion is extremely robust and precise where the robots are able to completely rotate the ball several times with many instances of the fingers coming in and out of contact. Given that the algorithm is open loop, such a level of precision and robustness was surprising even to the authors. Movies of sample experimental results are available via the world wide web at `http://controls.ame.nd.edu/manip/`.

# 5 Conclusions

This paper presented an outline of a robotic manipulation planning technique to implement so-called "finger gaiting." It is based on nonholonomic motion planning techniques which have been extended by the authors to a class of discontinuous problems which the algorithm can accommodate any smooth object and finger tip parameterizations and arbitrary controllable kinematics of the manipulators. Simulation as well as experimental results were presented. Not presented, but a direct result from the authors' previous efforts is a proof that force closure can be maintained throughout the manipulation process. Interested readers are referred to [12, 7] for an applicable proof.

Future work includes adopting a vision-based robotic control method to "close the loop" to further enhance robustness and precision.. Additionally, a current requirement of the algorithm is that the object and finger tip be smoothly parameterized. Work to extend the algorithm to non-smooth objects will be the subject of a future publication.

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