Gait Controllability for Legged Robots

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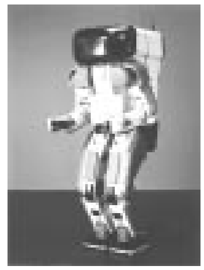
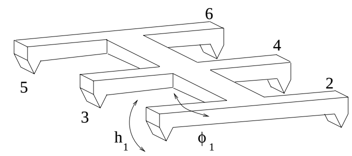
Outline:

1. Introduction and Background
2. Mathematical Preliminaries, Stratified Systems
3. Controllability
4. Motion Planning
5. Conclusions and Future Work

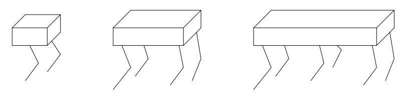
In this talk, we present a particular controllability test and a general trajectory generation scheme for a class of kinematic stratified system. Quasi–static robotic locomotion is a subset of such problems. The method does not depend upon the number of legs, nor is it based on foot placement concepts. Instead, our method is based on an extension of a nonlinear controllability test trajectory generation algorithm for smooth systems to the legged case, where the relevant mechanics are not smooth.

Introduction and Motivation





- We want to determine *general methods* for control and motion planning for such systems,
  - encompassing each type of problem,
  - spanning multiple morphologies.



- Ultimate goal $\Longrightarrow$ general theory for such systems.
- Main tool $\Longrightarrow$ idea of stratifications.
- Benefits $\Longrightarrow$ generalization, mechanical simplicity (underactuated systems, exploit nonlinearities).

This motivational slide is intended to illustrate that intermittent physical contact is a common phenomenon, which results in a problem with varying dimensionality. The changing dimensionality for the kinematic systems comes, at least for legged robot examples, from intermittent contact with some physical boundary. The picture of R2D2 and C3PO illustrates on way to characterize my work. There exist *general* for *smooth* nonholonomic systems (like R2D2), but not for nonsmooth systems (like C3PO). One main contribution of my thesis was extending some basic results for smooth to a class of nonsmooth systems we call *stratified*.

Stratified Control Systems

- Mathematical Definition: Partition of M into submanifolds that "meet nicely."
- On each stratum — different equations of motion.
- Restricted to each stratum: equation are smooth.
- Cyclic strata switches ⇒ Locomotion
- Control may need equations of motion in each *stratum*.

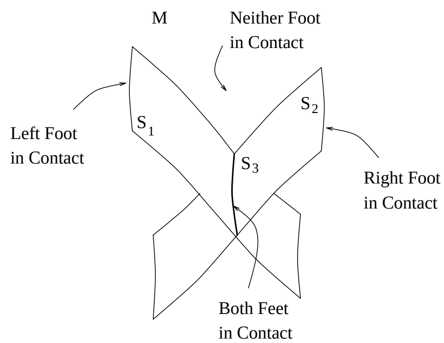


Figure 1. Stratified Configuration Space

If we take a close look at the structure of the configuration space of such systems we see that it is *stratified*, *i.e.*, there exist submanifolds of the configuration space on which the system has different equations of motion. A two–legged robot, for example, would have submanifolds of its configuration space corresponding to the its right foot being in contact with the ground. Similarly, there will be a different submanifold corresponding to its left foot being in contact with the ground. The intersection of these two submanifolds defines yet a third submanifold corresponding to both feet being in contact with the ground. From our common experience walking, we observe that locomotion results from following a path in configuration space which cyclically moves on and off each of these leaves, and without moving between at least some of the submanifolds, locomotion, and thus controllability would probably be impossible.

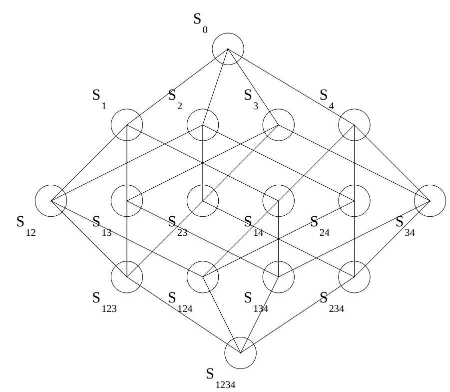
- Ability to locomote dependent on equations of motion in each submanifold.
- Transitioning among strata $\Longrightarrow$ non–smooth transitions.
- We will consider *driftless* control systems of the form:

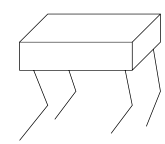
$$\dot{x} = g_1(x)u^1 + \cdots + g_m(x)u^m.$$

- Both the *form* and the *number* of g_i's may change in a non–smooth way between submanifolds – but are assumed smooth in each regime.

General Case:

- Multi–level stratifications are also possible.
  - nodes correspond to different strata
  - "allowable" connections determined by the kinematics of the system
- Geometric structure and algebraic (graph) structure.





- A *locomotive gait* = cyclic path in this graph.
- $\dot{x} = f(x) + g_{i,1}(x)u_1 + \cdots + g_{i,m_i}(x)u_{m_i}$ — with drift ($f(x)$ contains "momentum" terms).
- $\dot{x} = g_{i,1}(x)u_1 + \cdots + g_{i,m_i}(x)u_{m_i}$ — driftless.

This slide presents a slightly more complicated picture of our definition of a stratification. Since it is hard to think in more than three dimensions, an alternative way to view stratifications is by way of a graph structure.

Further pursuit of this algebraic approach will help link my work to that of the *hybrid* systems people.

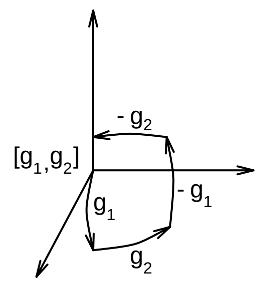
Some Standard Control Theory

- If the system

  $$\dot{x} = g_1(x)u_1 + \cdots + g_m(x)u_m$$

  is underactuated, linearization is *not* controllable.

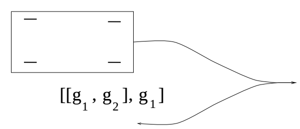
- Central to nonlinear control theory is the *Lie bracket*:



$$[g_1, g_2](x) = \frac{\partial g_2(x)}{\partial x} g_1(x) - \frac{\partial g_1(x)}{\partial x} g_2(x).$$

- Lie bracket motions generate *new directions* in which the system can move.

- Example: parallel parking.



This slide gives the definition of a Lie bracket. More importantly, it illustrates its central role in nonlinear control theory by showing how a "Lie bracket motion" is possible by appropriately modulating the control inputs. It is worth emphasizing that this Lie bracket motion only approximates the result if the system could actually flow along the Lie bracket vector field.

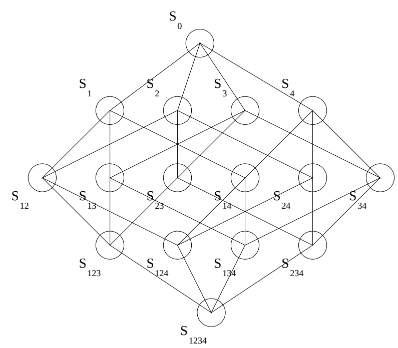
Controllability Tests:

- Equations of Motion + Lie brackets $\Longrightarrow$ reachable direction.
- All the directions the system can move is given by a *distribution*

$$\overline{\Delta} = \text{span}\{g_1, \ldots, g_m, [g_i, g_j], \ldots, [g_i, [g_j, g_k]], \ldots\}.$$

- **Theorem: (Chow)** $\dim\left(\overline{\Delta}\right) = \dim(M) \Longleftrightarrow$ STLC.

- But Chow's theorem only applies on *individual strata*.
- Stratified systems require a more global test.
- How to combine distributions defined on different strata?



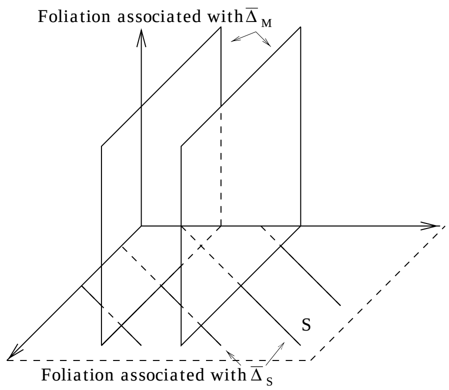
The object $\overline{\Delta}$ is called an *involutive distribution*. Frobenius' Theorem equates involutivity of a distribution with its integrability, thus justifying the statement regarding the existence of the surface (submanifold) F.

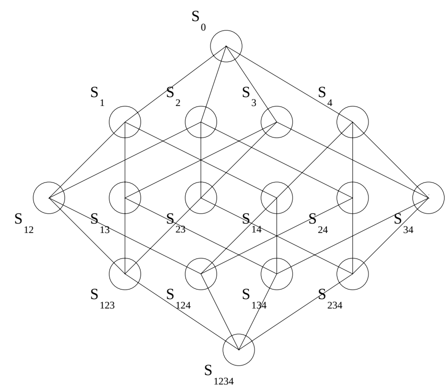
The intuition here is that the collection of all the Lie brackets between the control vector fields, and all the iterations thereof, gives us all the directions in which we can move. Logically, then, these directions are tangent to the surface that represents the set of reachable points for the system.

The problem for stratified systems is that these foliations are only defined on individual strata. Therefore, the problem is to determine the appropriate way to combine the foliations on multiple strata to determine controllability or do motion planning.

Stratified Controllability: Simple Case

- We can construct $\overline{\Delta}$ on each stratum of the configuration space.
- This defines the reachable directions on each stratum.
- Consider the relationship between only two strata:





- $\Longrightarrow$ some union $\bigcup_n \overline{\Delta}_n|_x$ describes controllable "directions."

Although Chow's theorem does not apply, it will be useful to compute the involutive closure of the control vector fields separately on each of the submanifolds present in the configuration space. This will define a foliation on each of the submanifolds.

Gait Controllability

- Define a gait as a ordered sequence of strata:

$$\mathcal{G} = \{S_{I_1}, S_{I_2}, \ldots, S_{I_n}, S_{I_{n+1}} = S_{I_1}\}.$$

- Define *gait controllable*: can reach an open neighborhood of initial point in bottom stratum.
- Construct the *gait distribution*:

$$\mathcal{D}_m = \sum_{i=2}^{m} \mathcal{D}_{i-1} + \left(\overline{\Delta}_{I_i} \cap TS_B\right).$$

- **Proposition:** If

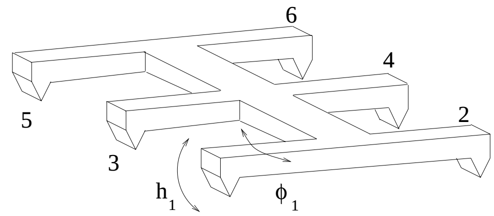
$$\dim\left(\mathcal{D}_m\right) = \dim\left(T_{x_0} S_B\right)$$

then the system is gait controllable from x_0.

- Proof: similar to Chow's theorem, construct the reachable set stratum by stratum around the gait.

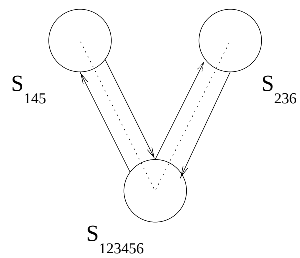
This is the main controllability result. Basically, if we project each distribution defined on each stratum in the gain onto the tangent space of the bottom stratum, then is is straight–forward to show that the system is gait controllable. The proof is just like Chow's theorem, but one must take care to "include" and "project" the allowable velocities into or onto higher or lower strata, respectively.

Hexapod Robot Example



- 2 degree of freedom legs (lift up and down, and swing forward and backward) $\Longrightarrow$ can not directly move sideways.

- First: assume a tripod gait.



$$
\begin{aligned}
\dot{x} &= l\cos\theta\,(\alpha(h_1)u_1 + \beta(h_2)u_2) \\
\dot{y} &= l\sin\theta\,(\alpha(h_1)u_1 + \beta(h_2)u_2) \\
\dot{\theta} &= l\alpha(h_1)u_1 - l\beta(h_2)u_2 \\
\dot{\phi}_1 &= u_1 \qquad \dot{\phi}_2 = u_2 \\
\dot{h}_1 &= u_3 \qquad \dot{h}_2 = u_4
\end{aligned}
$$

- All legs in contact: $\alpha = \beta = 1$, (or $\alpha = \beta = 0$).
- Legs 1,4,5 in contact: $\alpha = 1$, $\beta = 0$.
- Legs 2,3,6 in contact: $\alpha = 0$, $\beta = 1$.

We illustrate the trajectory generation algorithm using the same hexapod example from before.

Hexapod Controllability: Calculating distributions:

$$\overline{\Delta}_{123456} = \text{span}\left\{ \begin{pmatrix} l\cos\theta \\ l\sin\theta \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} l\cos\theta \\ l\sin\theta \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2l\sin\theta \\ 2l\cos\theta \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2l^2\cos\theta \\ 2l^2\sin\theta \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$\overline{\Delta}_{145} = \text{span}\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$\overline{\Delta}_{236} = \text{span}\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Since $\overline{\Delta}_B + \left(\overline{\Delta}_{145} \cap TS_B\right) + \left(\overline{\Delta}_{236} \cap TS_B\right) = TS_B$
$\Longrightarrow$ STLC on S_B.

Second: wave gait does not satisfy test.

These are simply the result of the calculations showing controllability for the hexapod robot.

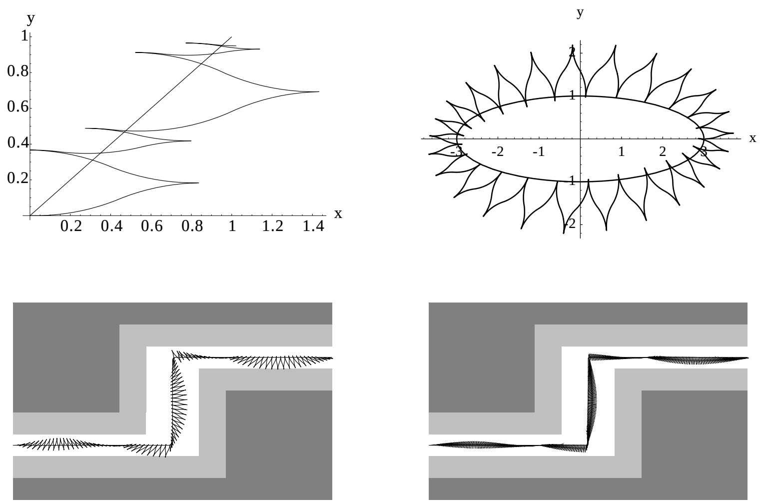
Stratified Motion Planning

- Controllability is a necessary condition to follow arbitrary trajectories.
- Stratified and Lie algebraic structure provides the mathematical foundation for our method.
- **Problem statement:** Determine control inputs (not foot placements) that steer system to final desired configuration.
- Our method modifies/extends that of Lafferriere and Sussmann.*
- Method based on a Campbell–Baker–Hausdorff expansion (Lie algebraic power series) of flows, and *strata decoupling*.
- *For controllable systems* arbitrary trajectories are possible. Gait stability and obstacle avoidance issues are naturally incorporated into the method.

*G. Lafferriere and Hector J. Sussmann. *A Differential Geometric Approach to Motion Planning*, Nonholonomic Motion Planning, 235–270, 1993.

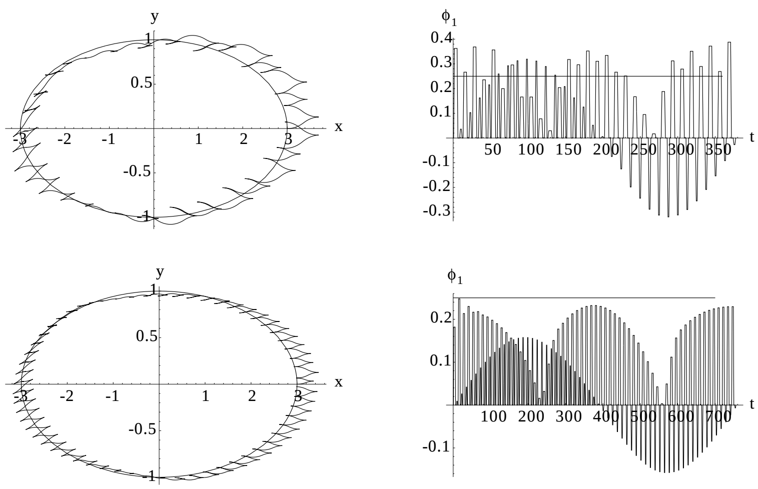
This is the introductory slide for the motion planning problem.

Example Trajectories:



This slide simply illustrates the path of the center of mass of the hexapod as at walks along a simple straight line path, as well as an elliptical path. Also, the bottom two figures illustrate the obstacle avoidance properties of this method. If a nominal trajectory avoids obstacles, then, for sufficiently small steps, the real trajectory avoids the obstacles as well.

Stability and Obstacle Avoidance



This slide simply illustrates the path of the center of mass of the hexapod as at walks along an elliptical path. We note, as would be intuitively obvious, and is easy to show, that if the robot takes smaller steps, it tracks the desired trajectory better.

Conclusions

- We have developed a controllability test for a large class of kinematic legged robotic system.
- The method is *independent of morphology*
- and general, *e.g.*, applies to grasping and cooperating robotic manipulation problems as well.
- We also highlighted our motion planning results.

Future Work

- Systems with *drift*.
- Closer connections with hybrid system theory.