

Guidelines for Plotting a Root Locus

For the characteristic equation

$$1 + KG(s) = 0,$$

assume that there are m zeros and n poles.

For $K > 0$:

1. Mark the poles of $G(s)$ with an $\times$ and the zeros with a $\circ$. We know that at $K = 0$, the locus starts at each pole, as $K \rightarrow +\infty$, one locus will terminate at each of the m zeros, and the rest $(n - m)$ will terminate at ∞ along the asymptotes.
2. Draw the locus on the real axis to the left of an odd number of poles plus zeros.
3. Draw the $n - m$ asymptotes. The angle of each asymptote will be

$$\alpha = \frac{180^\circ \pm k360^\circ}{n - m},$$

and the point where they intersect the real axis is at

$$\sigma_c = \frac{\sum_{i=1}^n p_i - \sum_{i=1}^m z_i}{n - m}.$$

4. Departure angles from the poles and arrival angles at the zeros that are not on the real axis are determined by selecting a point near the pole or zero and applying the angle condition ($180^\circ \pm k360^\circ$).
5. The point, σ_b at which the locus breaks away from the real axis (between two poles), or breaks into the real axis (between two zeros) is determined by finding the real values, $s = \sigma_b$ for which $\frac{dK}{ds} = 0$.

For $K < 0$:

1. Mark the poles of $G(s)$ with an $\times$ and the zeros with a $\circ$. We know that at $K = 0$, the locus starts at each pole, as $K \rightarrow +\infty$, one locus will terminate at each of the m zeros, and the rest $(n - m)$ will terminate at ∞ along the asymptotes.
2. Draw the locus on the real axis to the left of an *even* number of poles plus zeros.
3. Draw the $n - m$ asymptotes. The angle of each asymptote will be

$$\alpha = \frac{0^\circ \pm k360^\circ}{n - m},$$

and the point where they intersect the real axis is at

$$\sigma_c = \frac{\sum_{i=1}^n p_i - \sum_{i=1}^m z_i}{n - m}.$$

4. Departure angles from the poles and arrival angles at the zeros that are not on the real axis are determined by selecting a point near the pole or zero and applying the angle condition ($0^\circ \pm k360^\circ$).
5. The point, σ_b at which the locus breaks away from the real axis (between two poles), or breaks into the real axis (between two zeros) is determined by finding the real values, $s = \sigma_b$ for which $\frac{dK}{ds} = 0$.