University of Notre Dame
Aerospace and Mechanical Engineering

AME 437: Control Systems Engineering
Exam 1

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NAME: ______________________________

- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, your class notes, your own homework sets and any documents provided on the course homepage such as homework solutions, tables, etc.
- There are 3 questions. Problem 1 is worth 60 points, Problem 2 is worth 25 points and Problem 3 is worth 15 points.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- Do not start or turn the page until instructed to do so.

As with the tone-deaf, they don't know what they miss. They give a pitying chuckle at the news of scientists who have never read a major work of English literature. They dismiss them as ignorant specialists. Yet their own ignorance and their own specialization is just as startling. A good many times I have been present at gatherings of people who, by the standards of the traditional culture, are thought highly educated and who have with considerable gusto been expressing their incredulity at the illiteracy of scientists. Once or twice I have been provoked and have asked the company how many of them could describe the Second Law of Thermodynamics. The response was cold: it was also negative. Yet I was asking something which is about the scientific equivalent of: Have you read a work of Shakespeare's?

I now believe that if I had asked an even simpler question – such as, What do you mean by mass, or acceleration, which is the scientific equivalent of saying, Can you read? – not more than one in ten of the highly educated would have felt that I was speaking the same language. So the great edifice of modern physics goes up, and the majority of the cleverest people in the western world have about as much insight into it as their neolithic ancestors would have had.

— C. P. Snow, The Two Cultures.

1. It is now the year 2003 and you work for Hewlett-Packard developing their new line of ink jet printers. The basic mechanism for moving the printer cartridge across a sheet of paper is illustrated in Figure 1, where a voltage source is connected to a DC motor by the circuit illustrated, and the motor is attached to a light pulley, which is attached to a timing belt to which the printer cartridge assembly is attached.

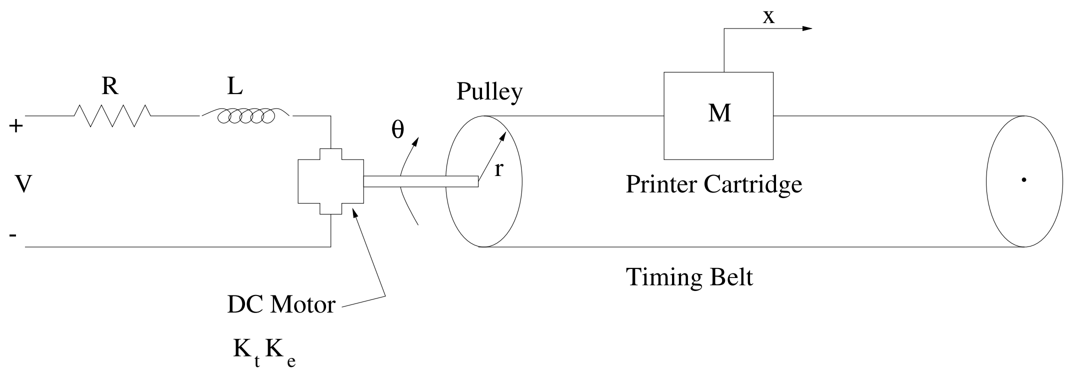


Figure 1. Printer Cartridge Control Problem.

Assume that the pulley is light, so that it has negligible inertia and that the timing belt does not stretch, so that you can ignore any elastic effects (and also $x = r\theta$). Also assume that the DC motor is has a torque constant, K_t, and a back emf constant K_e.

Find the transfer function from $V(s)$ to $X(s)$, *i.e.*, $\frac{X(S)}{V(S)}$. (20 points)

2. A more complete model of the printer system is illustrated in Figure 2. A colleague developed a block diagram representation for the system, illustrated in Figure 3 which now also includes the inertia of the pulleys and elastic effects in the timing belt. The output of this system, $Y(s)$ is the cartridge velocity, $R(s)$ is the desired, reference velocity, and $W(s)$ is a disturbance, used to model, for example, the effect of friction by the paper opposing the motion of the cartridge.

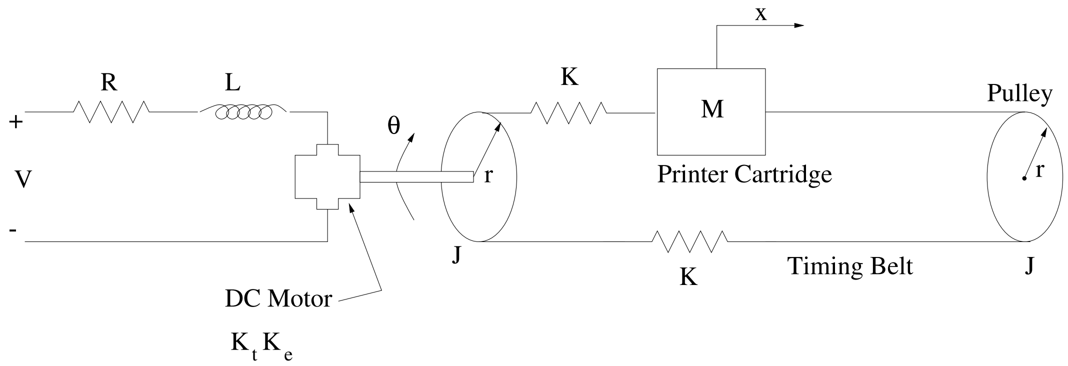


Figure 2. Printer Cartridge Control Problem.

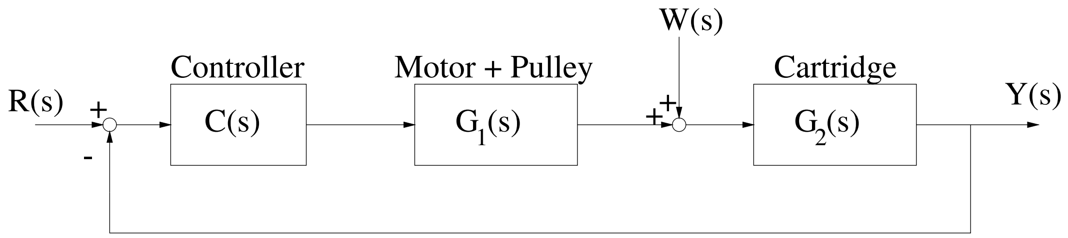


Figure 3. Printer Cartridge Speed Control Block Diagram.

(a) Find the transfer functions from $R(s)$ to $Y(s)$ and $W(s)$ to $Y(s)$, *i.e.*, express $Y(s)$ as

$$Y(s) = (\qquad) R(s) + (\qquad) W(s),$$

where the transfer functions appear between the parentheses. (20 points)

(b) Let

$$G_1(s) = \frac{5}{s^4 + 3s^3 + 8s^2 + 7s + 5} \qquad \text{and} \qquad G_2 = \frac{10}{s+10},$$

and let $C(s) = K_p$ (proportional control).

i. Compute the steady state *error* to a unit step input in the reference input, *i.e.*, $R(s) = \frac{1}{s}$. Assume there is no disturbance. (10 points)

ii. Compute the steady state *error* to a unit step input in the disturbance, *i.e.*, $W(s) = \frac{1}{s}$. (10 points)

iii. If $K_p = 10$ is the transfer function from $R(s)$ to $Y(s)$ stable? (10 points)

3. A friend tells you the following:

> The Laplace transform of the derivative of a function is s times the Laplace transform of the original function. We do this all the time when we take the Laplace transform of a differential equation:
>
> $$\ddot{y}(t) + \dot{y}(t) + y(t) = 0 \Longleftrightarrow Y(s)(s^2 + s + 1) = 0.$$
>
> So, consider $\cos(t)$. Obviously, $\frac{d}{dt}\cos(t) = -\sin(t)$. However, using the table, we see that
>
> $$\mathfrak{L}\left[\frac{d}{dt}\cos(t)\right] = s\mathfrak{L}\left[\cos(t)\right] = s\frac{s}{s^2+1} = \frac{s^2}{s^2+1}$$
>
> But looking at the table, we have
>
> $$\mathfrak{L}\left[-\sin(t)\right] = -\frac{1}{s^2+1},$$
>
> which is not the same thing.

Explain what was wrong about your friend's reasoning. (5 points)

4. Consider the following block diagram:

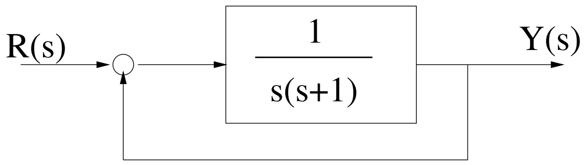
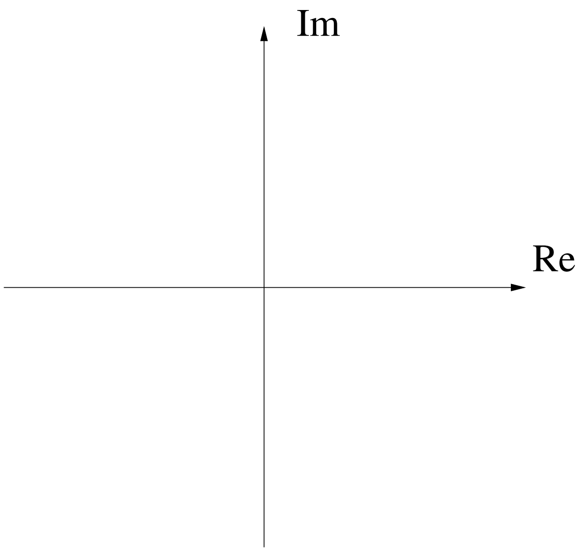
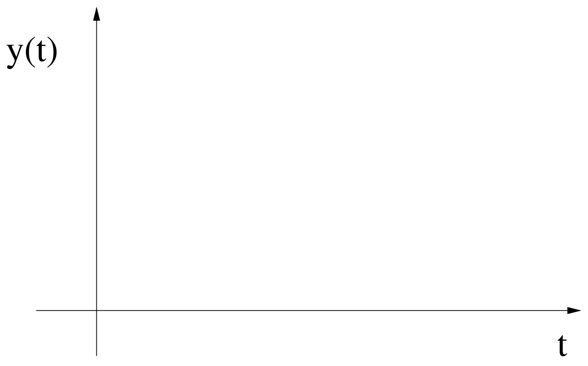


Figure 4. Block Diagram for Problem 4.

(a) On the s-plan, plot the location of the closed loop poles. (5 point)



(b) Sketch what the response of the system would look like to a unit step reference input. (You do not have to analytically solve the system – just a rough sketch based upon the pole locations is fine). (5 points)



(c) On the same plot, sketch the response if there is an additional zero added to the transfer function at $s = -100$. Be sure to label which response is for which question. (5 points)

(d) On the same plot, sketch the response if there is an additional zero added to the transfer function at $s = -1$. (5 points)

(e) On the same plot, sketch the response if there is an additional pole added to the transfer function at $s = +1$. (5 points)