University of Notre Dame
Aerospace and Mechanical Engineering

AME 437: Control Systems Engineering
Homework 3

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Due: never

Problems

1. Consider the transfer function

$$G(s) = \frac{55}{s^3 + 6s^2 + 15s + 20}.$$

   (a) Determine the gain and phase margins of this transfer function in a unity feedback configuration.

   (b) Design a lead compensator so that the unity feedback system has a damping ratio of at least $\zeta = 0.15$.

2. Consider

$$G(s) = \frac{40}{s(s+2)}.$$

   (a) Determine the unity feedback phase margin.

   (b) Design a lead compensator so that the phase margin is at lease 45°.

3. Consider the Bode plot illustrated in Figure 1.

   (a) What would be the steady state error of the unity feedback system to a step input?

   (b) What would be the steady state error of the unity feedback system to a ramp input?

4. Consider the Bode plot illustrated in Figure 2.

   (a) What would be the steady state error of the unity feedback system to a step input?

   (b) What would be the steady state error of the unity feedback system to a ramp input?

Solutions

1. (a) The Bode plot of the system is illustrated in Figure 3. The phase margin is 9.7°, which, from the chart handed out in class, corresponds to a damping ratio of $\zeta \approx 0.09$. The crossover frequency is 3.5 rad/sec.

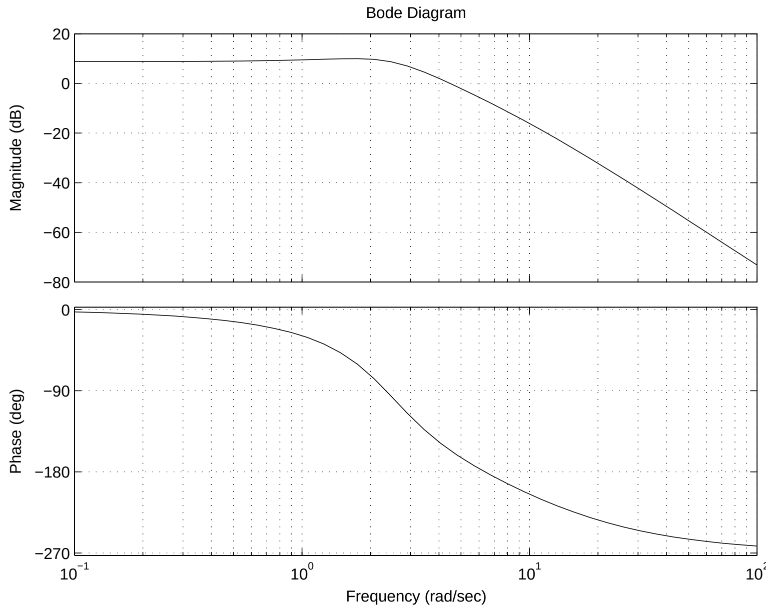

Figure 1. Bode plot for system in Problem 3.

(b) From the chart in class, a 15° phase margin corresponds to the desired damping ratio. In order to assure that we can add the necessary 6° of phase margin, we pick $\frac{1}{\alpha} = 4$, which corresponds to adding 35° of additional phase). We'll see that this is basically barely adequate. Since we want to place the maximum phase at $\omega = 3.5$, we solve

$$\omega\text{max} = \frac{1}{T\sqrt{\alpha}} \qquad \Longrightarrow \qquad T = \frac{1}{\omega\text{max}\sqrt{\alpha}}$$

for T. Thus, $T = .57$. Thus, the compensator is of the form

$$C(s) = \frac{Ts+1}{\alpha Ts+1} = \frac{.57s+1}{.14s+1}.$$

The Bode plot of the compensated system ($C(s)G(s)$) is illustrated in Figure 4, and the Bode plot for both are illustrated in Figure 5.

The reason that we need to pick α to add much more phase is that the compensator also shifts the crossover frequency, as can be seen in Figure 5, and we compute T to have the maximum phase lead of the compensator to be at the original, uncompensated, crossover frequency. If we needed to iterate the design, we could observe that the new crossover is at 5 rad/sec. Using that value, we find $T = 0.4$. Using that in a compensator, we have

$$C(s) = \frac{.4s+1}{.1s+1}.$$

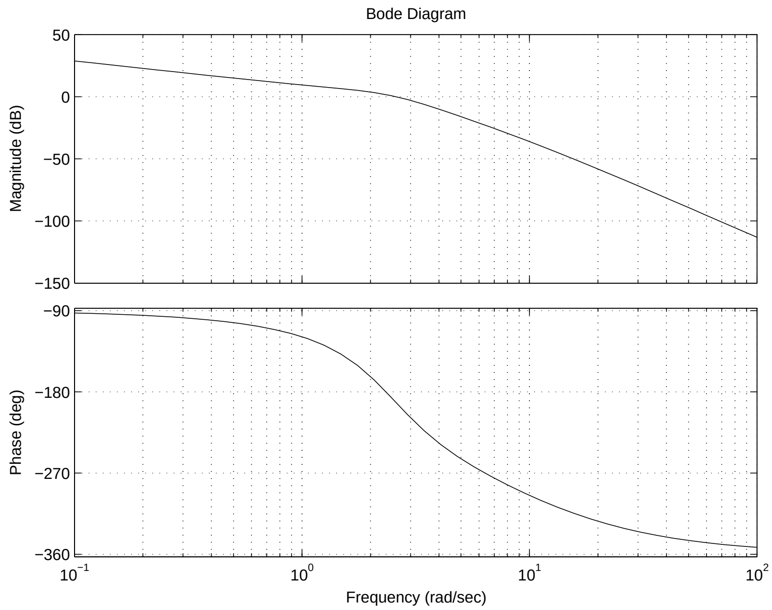

Figure 2. Bode plot for system in Problem 4.

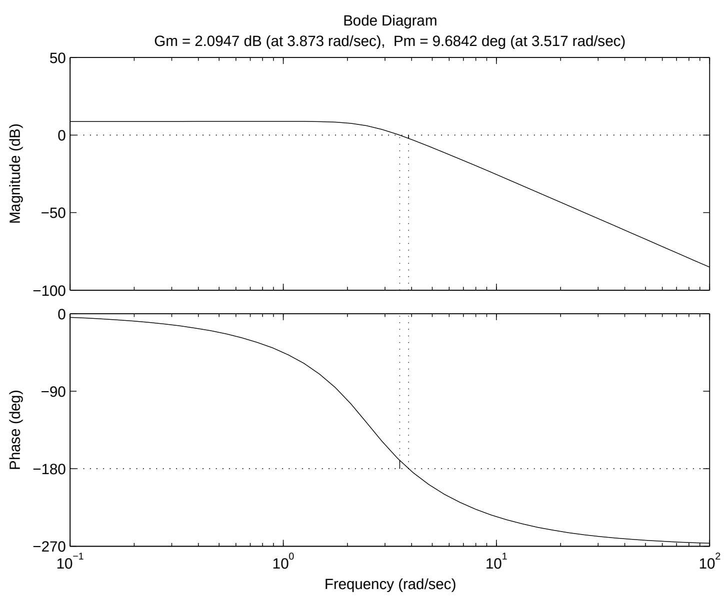

Figure 3. Bode plot for uncompensated system in Problem 1.

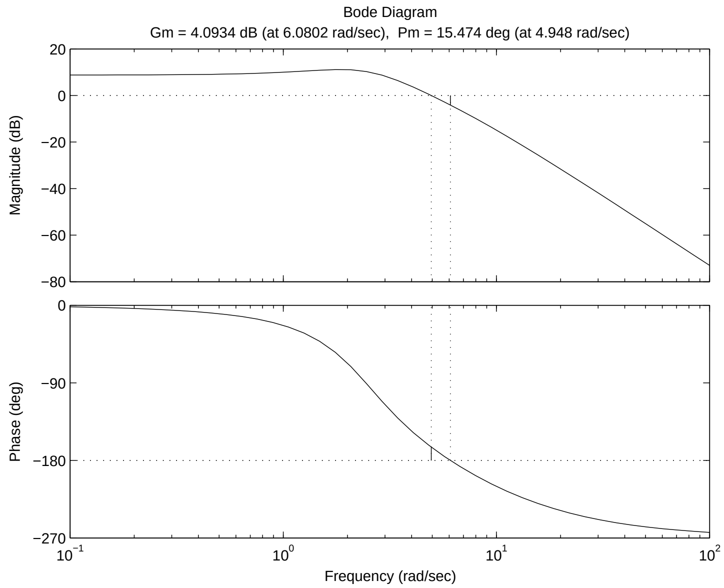

Figure 4. Bode plot for compensated system in Problem 1.

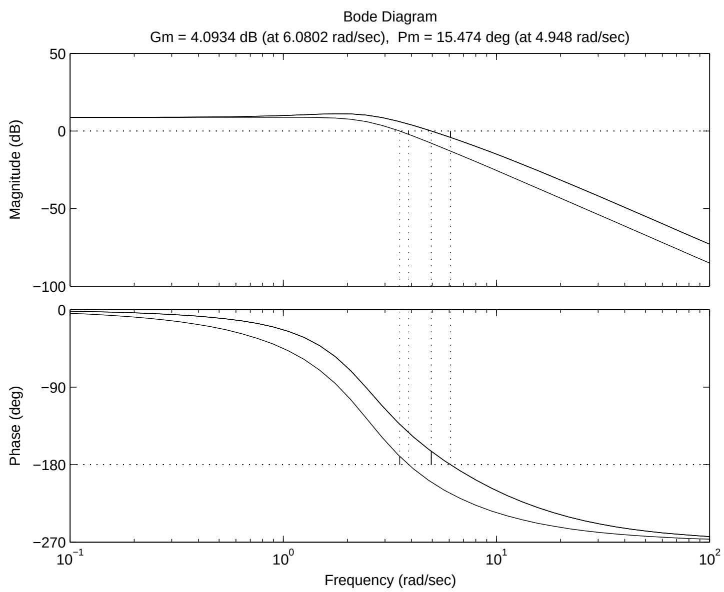

Figure 5. Bode plot for both systems in Problem 1.

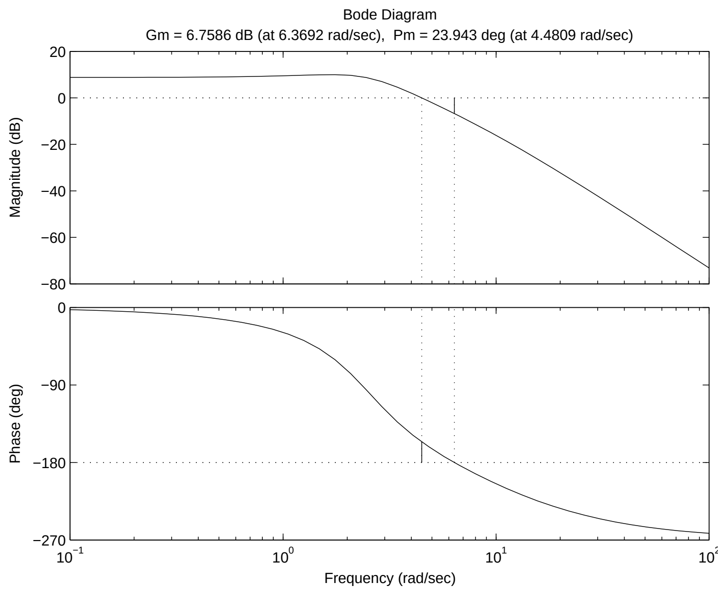

Figure 6. Bode plot for iterated design for Problem 1.

The Bode plot using that compensator is illustrated in Figure 6, as you can see that the phase margin is increased to 23°.

2. (a) The Bode plot for the uncompensated system is illustrated in Figure 7. The system has a phase margin of approximately 18°. The crossover frequency is 6.2 rad/sec.

   (b) We need to add at least 27° degrees of phase margin. We will pick $\frac{1}{\alpha} = 10$, which attempts to add about double that. Note that

$$T = \frac{1}{6.2\sqrt{.1}} \approx 0.5.$$

Thus

$$C(s) = \frac{0.5s + 1}{0.05s + 1}$$

and the Bode plot for the compensated system is illustrated in Figure 8. Both Bode plots are illustrated in Figure 9.

3. (a) The slope of the magnitude plot for low frequencies is 0, so this is a type 0 system. The value at low frequencies (after zooming in) is approximately 8.8. Thus

$$20 \log K_p = 8.8 \qquad \Longrightarrow \qquad K_p = 2.75 \qquad \Longrightarrow \qquad e_{ss} = \frac{1}{1 + 2.75} = 0.2664.$$

   (b) ∞.

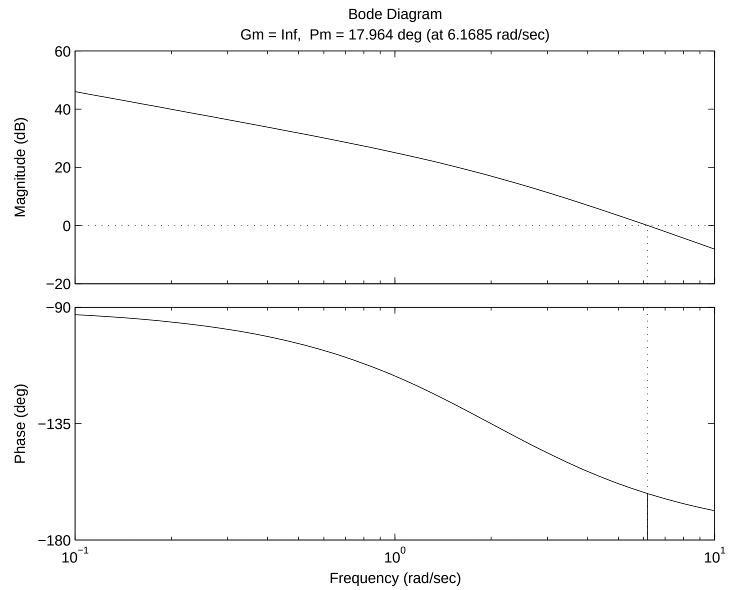

Figure 7. Bode plot for uncompensated system in Problem 2.

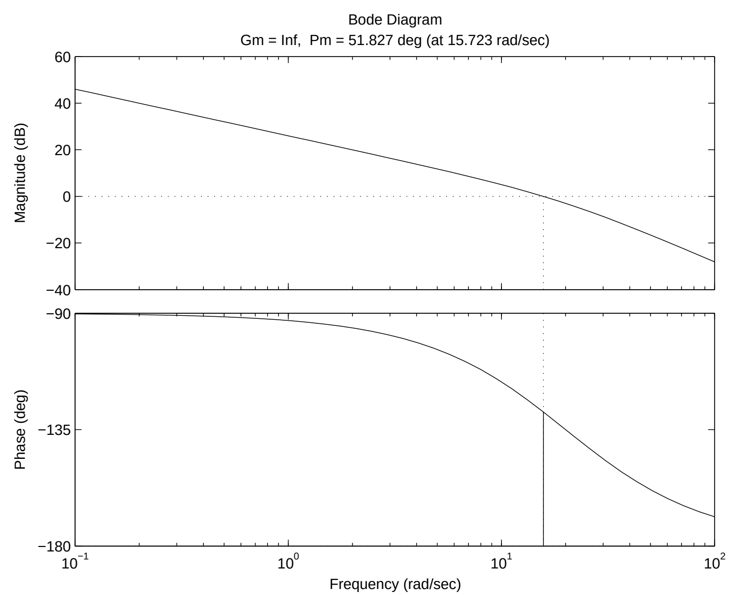

Figure 8. Bode plot for compensated system in Problem 2.

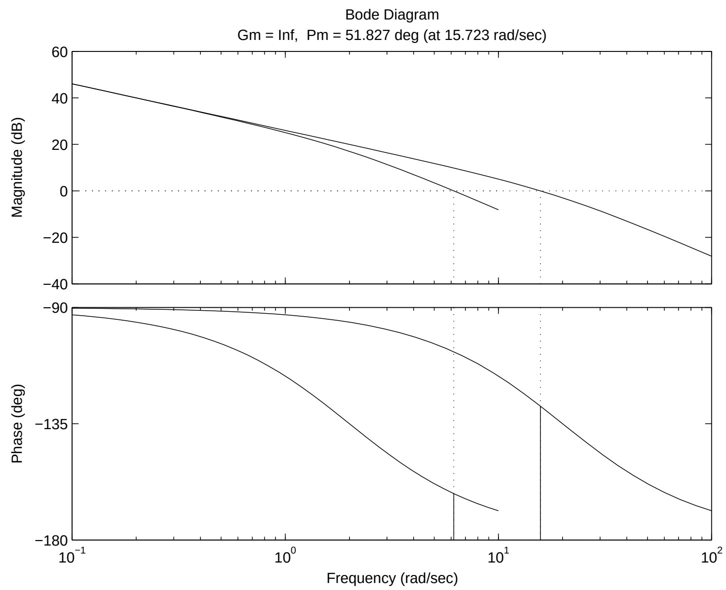

Figure 9. Bode plot for both systems in Problem 2.

4. (a) The slop of the magnitude plot for low frequencies is -20 dB/decade. Thus it is a type 1 system. Thus, for a step, $e_{ss} = 0$.

   (b) The low frequency asymptote passes through the value of 9.6 at $\omega = 1$. Thus,

$$20 \log K_v = 9.6 \qquad \Longrightarrow \qquad K_v = 3.02 \qquad \Longrightarrow \qquad e_{ss} = 0.33.$$