2.9.1.2 Find the Laplace transform by applying the definition of the transform

$$f(t)=\begin{cases}1 & T\le t\le 2T\\ 0 & \text{otherwise}\end{cases}$$

Solution

$$F(s)=\int_0^{+\infty} f(t)e^{-st}dt=\int_0^T 0e^{-st}dt+\int_T^{2T}1e^{-st}dt+\int_{2T}^{\infty}0e^{-st}dt$$

$$=\int_T^{2T}e^{-st}dt=-\frac{1}{s}e^{-st}\Big|_T^{2T}=\frac{1}{s}(e^{-sT}-e^{-2sT})$$

2.9.2.1 Express the function as a linear combination of simple functions and then find its Laplace transform

$$f(t)=\begin{cases}t & 0\le t\le T\\ T & T<t\le 2T\\ 0 & \text{otherwise}\end{cases}$$

Solution

$$f(t)=t[1(t)-1(t-T)]+T[1(t-T)-1(t-2T)]$$

$$=t1(t)-(t-T)1(t-T)-T1(t-2T)$$

$$F(s)=\frac{1}{s^2}-\frac{e^{-sT}}{s^2}-\frac{T}{s}e^{-2sT}\cancel{+\frac{T}{s}e^{-sT}}$$

2.9.3.1 Find the Laplace transform of the function $\cos\omega t$. Assume the functions are zero for $t<0$

$$\mathcal{L}\{\cos\omega t\}=\int_0^{\infty}\cos\omega t\, e^{-st}dt=\int_0^{\infty}\frac{e^{-st}}{\omega}d\sin\omega t=\frac{1}{\omega}\sin\omega t\, e^{-st}\Big|_0^{\infty}+\frac{s}{\omega}\int_0^{\infty}\sin\omega t\, e^{-st}dt$$

$$=-\frac{s}{\omega^2}\int_0^{\infty}e^{-st}d\cos\omega t=-\frac{s}{\omega^2}\cos\omega t\, e^{-st}\Big|_0^{\infty}-\frac{s^2}{\omega^2}\int_0^{\infty}\cos\omega t\, e^{-st}dt$$

$$=\frac{s}{\omega^2}-\frac{s^2}{\omega^2}\mathcal{L}\{\cos\omega t\}$$

$$\left(1+\frac{s^2}{\omega^2}\right)\mathcal{L}\{\cos\omega t\}=\frac{s}{\omega^2}\qquad \mathcal{L}\{\cos\omega t\}=\frac{s}{\omega^2}\cdot\frac{\omega^2}{s^2+\omega^2}=\frac{s}{s^2+\omega^2}$$