2.9.5 Solve each differential equation for the specified initial conditions using Laplace transform

2 $y^{[2]}(t) - 2y^{[1]}(t) - 3y(t) = 0, \quad y(0) = 1, \quad y'(0) = 7$

Solution

$$[s^2Y(s) - sy(0) - y'(0)] - 2[sY(s) - y(0)] - 3Y(s) = 0$$

$$(s^2 - 2s - 3)Y(s) = sy(0) + y'(0) - 2y(0) = s\times1 + 7 - 2\times1 = s + 5$$

$$Y(s) = \frac{s+5}{s^2-2s-3} = \frac{s+5}{(s+1)(s-3)} = \frac{2}{s-3} - \frac{1}{s+1}$$

$$\boxed{y(t) = 2e^{3t} - e^{-t}}$$

4 $y^{[2]}(t) + 2y^{[1]}(t) - 3y(t) = 0, \quad y(0) = 0 \quad y^{[1]}(0) = 4$

Solution $\quad s^2Y(s) - sy(0) - y'(0) + 2[sY(s) - y(0)] - 3Y(s) = 0$

$$(s^2 + 2s - 3)Y(s) = sy(0) + y'(0) + 2y(0) = 4$$

$$Y(s) = \frac{4}{s^2+2s-3} = \frac{1}{s-1} - \frac{1}{s+3}$$

DSlove

$$y(t) = e^{t} - e^{-3t}$$

6 $y^{[2]}(t) - 4y^{[1]}(t) + 5y(t) = 0 \quad y(0) = 1 \quad y^{[1]}(0) = 3$

Solution $\quad [s^2Y(s) - sy(0) - y'(0)] - 4[sY(s) - y(0)] + 5Y(s) = 0$

$$(s^2 - 4s + 5)Y(s) = sy(0) + y'(0) - 4y(0) = s - 1$$

$$Y(s) = \frac{s-1}{s^2-4s+5} = \frac{M}{s-2-j} + \frac{M^*}{s-2+j}$$

$$M = (s-2-j)Y(s)\Big|_{s=2+j} = \frac{s-1}{s-2-j}\Big|_{s=2+j} = \frac{1}{2}(1-j) = \frac{\sqrt{2}}{2}\angle{-0.7854r}$$

$$Y(s) = \frac{\frac{\sqrt{2}}{2}e^{-j0.7854}}{s-2-j} + \frac{\frac{\sqrt{2}}{2}e^{j0.7854}}{s-2+j}$$

$$y(t) = 2\times\frac{\sqrt{2}}{2}e^{2t}\cos(t-0.7854) = \sqrt{2}e^{2t}\cos(t-0.7854) = (\tfrac{1}{2}+\tfrac{i}{2})e^{(2-i)t}(1-ie^{2it})$$

8 $y^{[2]}(t) - 4y^{[1]}(t) + y(t) = 0 \quad y(0) = 1 \quad y^{[1]}(0) = -2$

Solution $\quad [s^2Y(s) - sy(0) - y'(0)] - 4[sY(s) - y(0)] + Y(s) = 0$

$$(s^2 - 4s + 1)Y(s) = sy(0) + y'(0) - 4y(0) = s - 2 - 4 = s - 6$$

$$Y(s) = \frac{s-6}{s^2-4s+1} = \frac{s-6}{[s-(2+\sqrt{3})][s-(2-\sqrt{3})]} = \frac{3-4\sqrt{3}}{6[s-(2+\sqrt{3})]} + \frac{4\sqrt{3}+3}{6[s-(2-\sqrt{3})]}$$

$$\boxed{y(t) = \frac{3-4\sqrt{3}}{6}e^{(2+\sqrt{3})t} + \frac{3+4\sqrt{3}}{6}e^{(2-\sqrt{3})t}}$$