University of Notre Dame
Aerospace and Mechanical Engineering

AME 437: Control Systems Engineering
Homework 1 Solutions

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1. From Newton's law, we know that $I\ddot{\theta}$ is equal to the sum of the moments. Therefore,
$$I\ddot{\theta} = \tau - mgl\cos\theta.$$
For small θ, $\cos\theta \approx 1$. Thus
$$I\ddot{\theta} = \tau - mgl.$$
Letting $I = 1$ and $mgl = 1$, the equation of motion is
$$\ddot{\theta} = \tau - 1.$$
Converting to two first order equations where $x_1 = \theta$ and $x_2 = \dot{\theta}$ gives
$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \tau - 1 \end{bmatrix}.$$

2. Implementing a PID controller:

   (a) For proportional control,
   $$\tau = k_p(\theta_d - \theta).$$
   Using `ode45()` in Matlab the following is saved as pid.m:

```
%
%   The is the 'odefile' called by ode45 to solve the differential
%   equations.
%
%   I called this file 'pid.m'
%
%   B. Goodwine, February 17, 2003.
%

function out = odefile(t,y,options,kp,kd,ki,td);
out = [y(2); -kd*y(2) - kp*y(1) + ki*y(3) + kp*td - 1; td - y(1)];
```

   i. In Matlab, letting

```
>> kp = 5;
>> kd = 0;
>> ki = 0;
>> td = 1;
>> options = odeset('OutputFcn','odeplot','Outputsel',[1]);
>> [t y] = ode45('pid', [0 20], [0 0 0],options,kp,kd,ki,td);
```

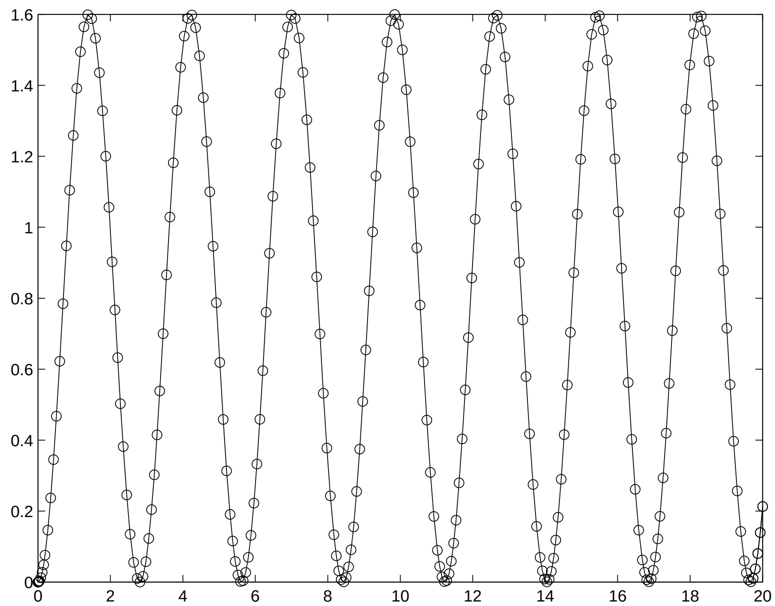

Figure 1: Response for proportional control where $k_p = 5$.

produced the result in Figure 1. Clearly the response is oscillatory and the average value is approximately $x_1 = 0.8$.

ii. Increasing $k_p = 10$ produced the result in Figure 2. In this case the solution is still oscillatory, but the average value has increased to $x_1 = 0.9$ and the frequency has increased. Since the mean steady state value has increased from 0.8 to 0.9, the steady state error for $\theta_d = 1$ has decreased.

(b) Adding derivative control introduces damping into the response.

i. Now we let $k_p = 5$ and $k_d = 0.5$. The response is illustrated in Figure 3.

ii. Increasing $k_d = 1$, produces the response in Figure 4, which has a reduced settling time compared to the case where $k_d = 0.5$.

iii. Increasing $k_d = 10$, produces the response in Figure 5, which displays no oscillatory response.

iv. Increasing $k_p = 10$ and keeping $k_d = 10$ produces the response in Figure 6, which has decreased steady state error.

v. Figure 6 also illustrates that the rise time decreases if k_p is increased.

(c) Adding integral control eliminates the steady state error, so $\theta_d \to 1$ as $t \to \infty$..

i. For $k_p = 1$, $k_d = 1$ and $k_i = 0.5$, the response of the robot arm is illustrated in Figure 7. There is no steady state error.

ii. Increasing $k_i = 0.75$, produces the response illustrated in Figure 8, which displays increased overshoot and settling time.

iii. Letting $k_p = 10$, $k_d = 1$ and $k_i = 0.5$, compared to 7 for problem 2(c)i, Figure 9 illustrates that overshoot is increased from approximately 25% to approximately 50%.

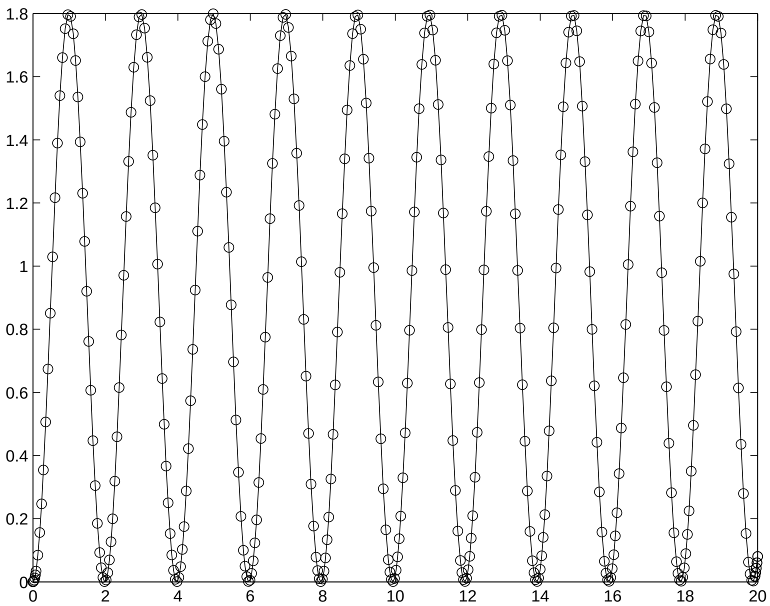

Figure 2: Response for proportional control where $k_p = 10$.

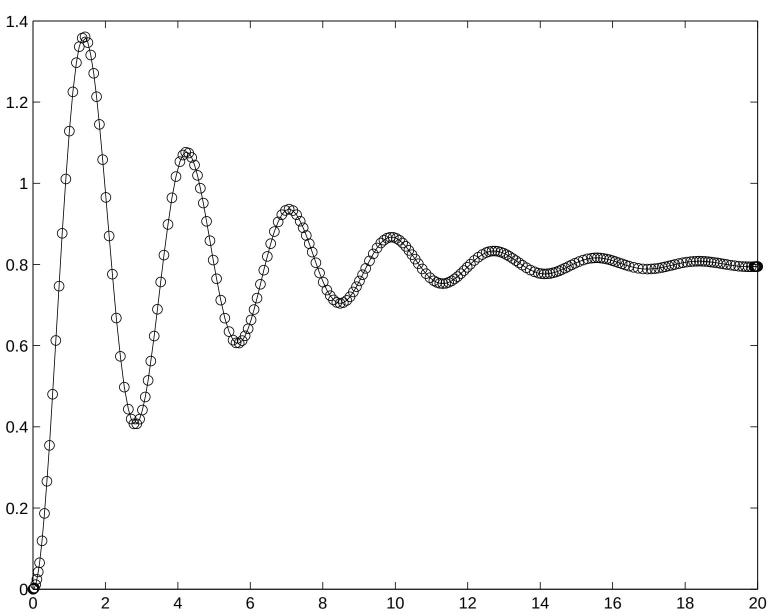

Figure 3: Response for proportional plus derivative control where $k_p = 5$ and $k_d = 0.5$.

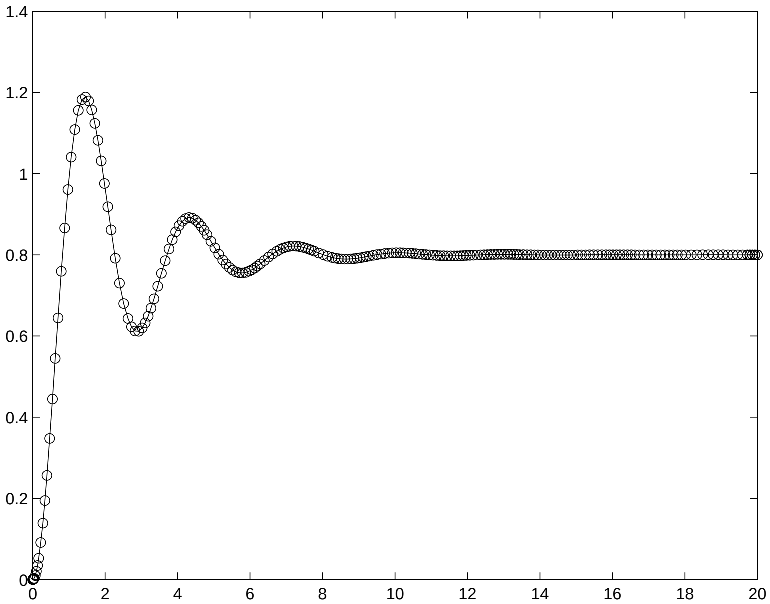

Figure 4: Response for proportional plus derivative control where $k_p = 5$ and $k_d = 1$.

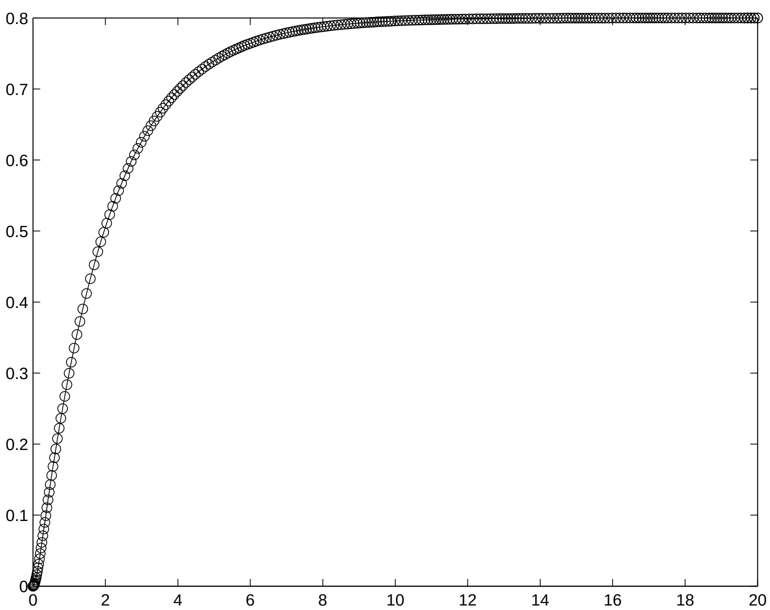

Figure 5: Response for proportional plus derivative control where $k_p = 5$ and $k_d = 10$.

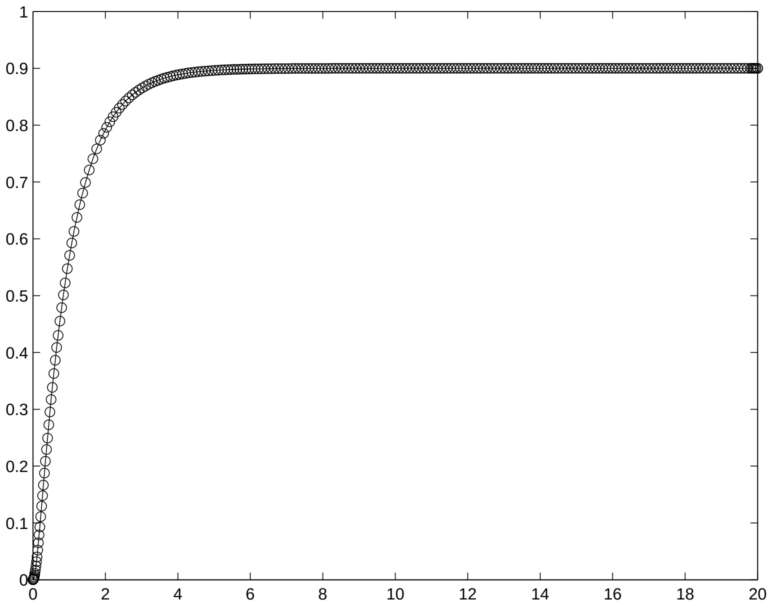

Figure 6: Response for proportional plus derivative control where $k_p = 10$ and $k_d = 1$.

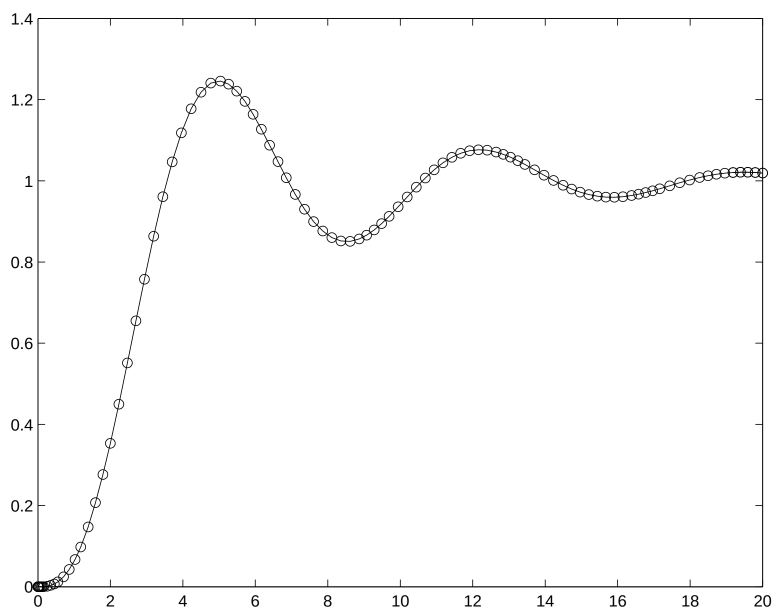

Figure 7: Response for proportional plus derivative plus integral control where $k_p = 1$, $k_d = 1$ and $k_i = 0.5$.

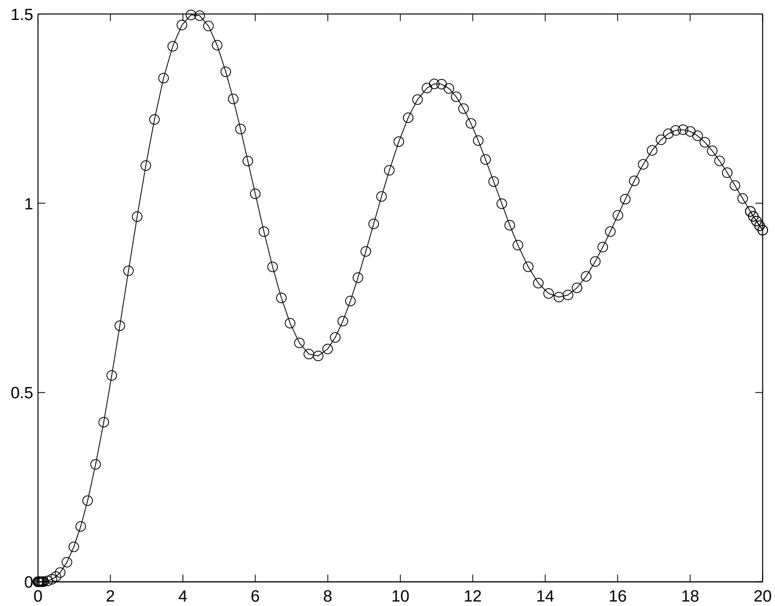

Figure 8: Response for proportional plus derivative plus integral control where $k_p = 1$, $k_d = 1$ and $k_i = 0.75$.

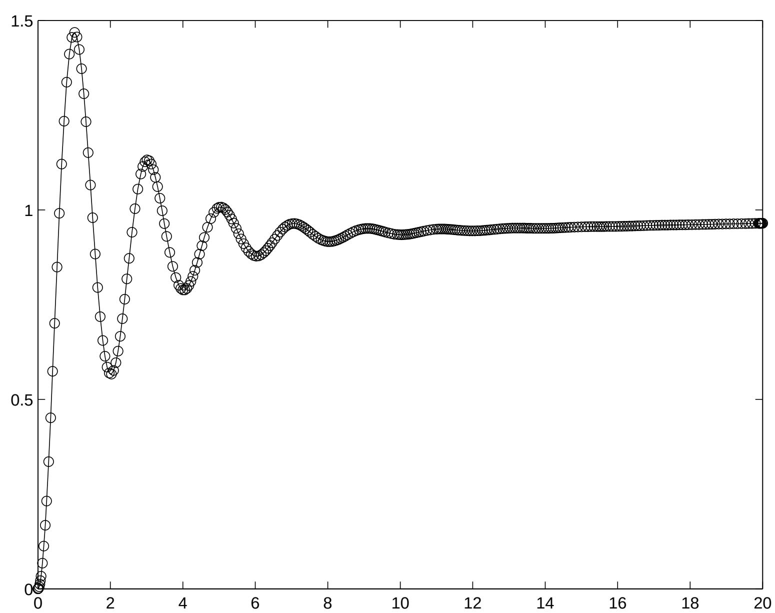

Figure 9: Response for proportional plus derivative plus integral control where $k_p = 10$, $k_d = 1$ and $k_i = 0.5$.