University of Notre Dame
Aerospace and Mechanical Engineering

AME 437: Control Systems Engineering
Homework 4 Solutions

B. Goodwine
February 17, 2003

1. (4.6.1:5) If

$$GH(s) = \frac{K(s+1)}{s(s+4)(s+8)},$$

The characteristic polynomial is

$$1 + \frac{K(s+1)}{s(s+4)(s+8)} = 0,$$

or

$$\begin{aligned} s(s+4)(s+8) + K(s+1) &= 0 \\ s^3 + 12s^2 + (32+K)s + K &= 0. \end{aligned}$$

You can use either the `roots()` command or `pzmap()` in Matlab to compute the roots for various values of K. Using `roots()`, you would have to plot the values by hand. Figure 1 was created by the following sequence of Matlab commands.

```
>> pzmap(1,[1 12 32.1 .1])
>> hold on
>> pzmap(1,[1 12 32.5 .5])
>> pzmap(1,[1 12 33 1])
>> pzmap(1,[1 12 33.5 1.5])
>> pzmap(1,[1 12 33.8 1.8])
>> pzmap(1,[1 12 34 2])
>> pzmap(1,[1 12 35 3])
>> pzmap(1,[1 12 36 4])
>> pzmap(1,[1 12 52 20])
>> pzmap(1,[1 12 82 50])
```

Connecting the roots with a line would produce Figure 2.

2. (4.6.1:6) Since

$$GH(s) = \frac{K}{(s+1)(s+10)},$$

the characteristic equation is

$$s^2 + 11s + (10+K) = 0.$$

Using `pzmap()` again:

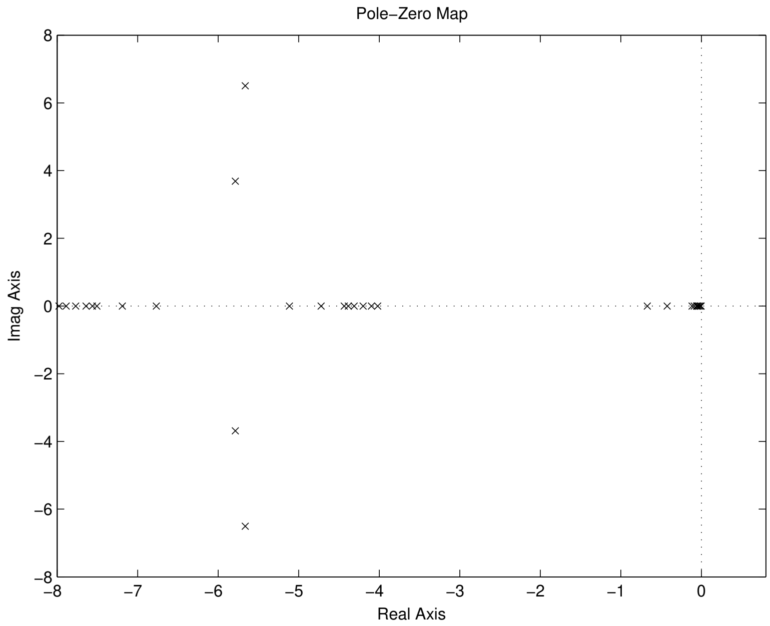

Figure 1. Roots for various K for problem 4.6.1:5.

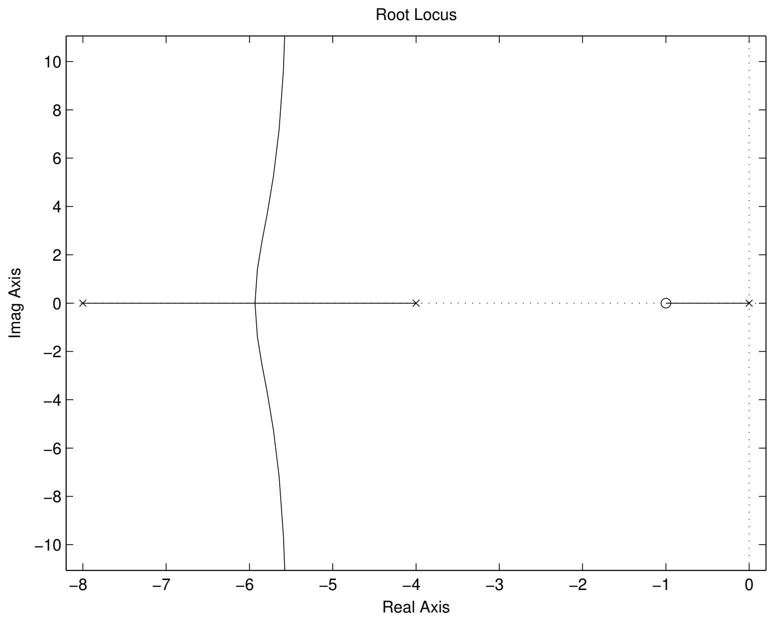

Figure 2. Connecting the roots for various K for problem 4.6.1:5.

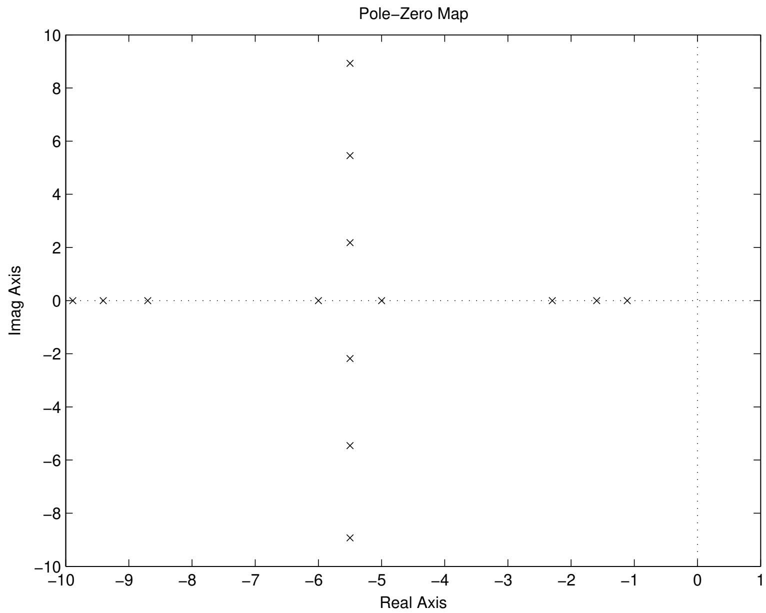

Figure 3. Roots for various K for problem 4.6.1:6.

```
>> pzmap(1,[1 11 11])
>> hold on
>> pzmap(1,[1 11 15])
>> pzmap(1,[1 11 20])
>> pzmap(1,[1 11 30])
>> pzmap(1,[1 11 35])
>> pzmap(1,[1 11 60])
>> pzmap(1,[1 11 110])
```

produces Figure 3. Connecting the dots produces Figure 4.

3. (4.6.2:9) The Routh array is:

$$\begin{array}{c|ccc} s^4 & 1 & 196 & 640 \\ s^3 & 24 & 624 & 0 \\ s^2 & 170 & 640 & \\ s^1 & 533.65 & & \\ s^0 & 640 & & \end{array}.$$

   Since there are no sign changes, there are no RHP poles.

4. (4.6.2:11) Was changed to extra credit.

5. (4.6.3:3) Since

$$GH(s) = \frac{K(s+1)}{(s-1)(s-2)},$$

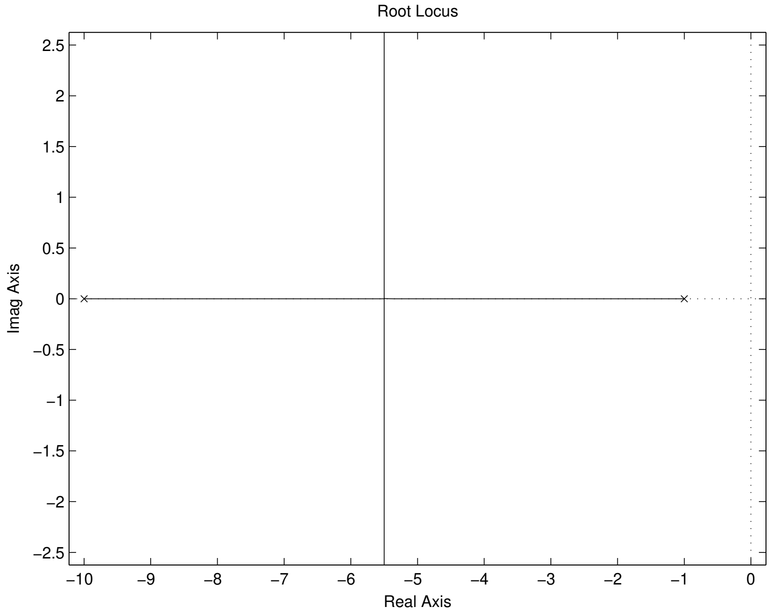

Figure 4. Connecting the roots for various K for problem 4.6.1:6.

the characteristic equation is

$$s^2 - 3s + 2 + Ks + K = s^2 + (K-3)s + (K+2) = 0.$$

The Routh array is

$$\begin{array}{c|cc} s^2 & 1 & K+2 \\ s^1 & K-3 & 0 \\ s^0 & K+2 & \end{array}.$$

So, $K > 3$ for stability.

6. (4.6.3:5) Since

$$GH(s) = \frac{K}{(s+3)^3},$$

the characteristic equation is

$$(s+3)^3 + K = s^3 + 9s^2 + 27S + (K+27) = 0.$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 27 \\ s^2 & 9 & K+27 \\ s^1 & 24 - \frac{K}{9} & 0 \\ s^0 & K+27 & \end{array}.$$

Thus, $-27 < K < 216$ for stability.

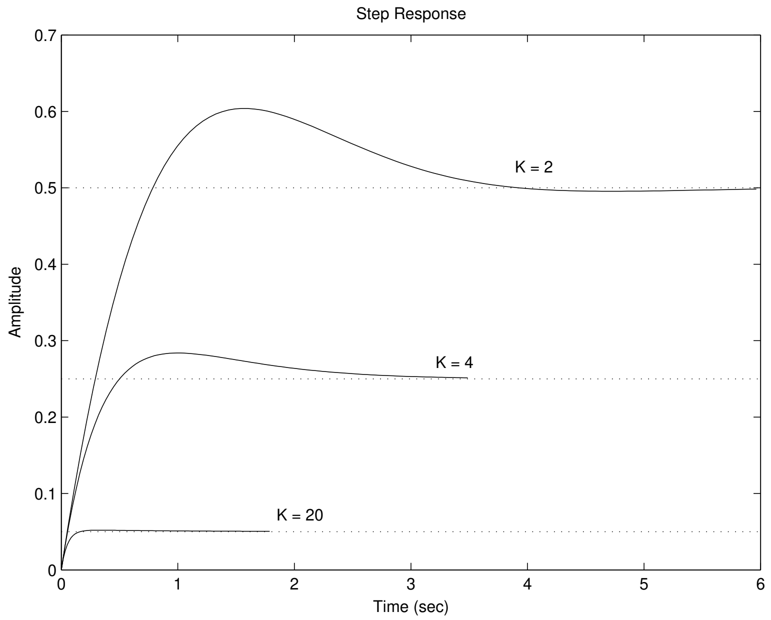

Figure 5. Step responses for $K = 2, 4, 20$ for problem 4.6.4:2.

7. (4.6.4:2) Since

$$G(s) = \frac{(s+1)}{s^2}$$

and $H = K$, The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{K(s+1)}{s^2 + K(s+1)}.$$

To find the closed loop step response, use the Matlab `step()` function. The following commands produce Figure 5.

```
>> K = 2;
>> num = [1 1];
>> den = [1 K K];
>> step(num,den);
>> hold on
>> K = 4;
>> den = [1 K K];
>> step(num,den);
>> K = 20;
>> den = [1 K K];
>> step(num,den);
```

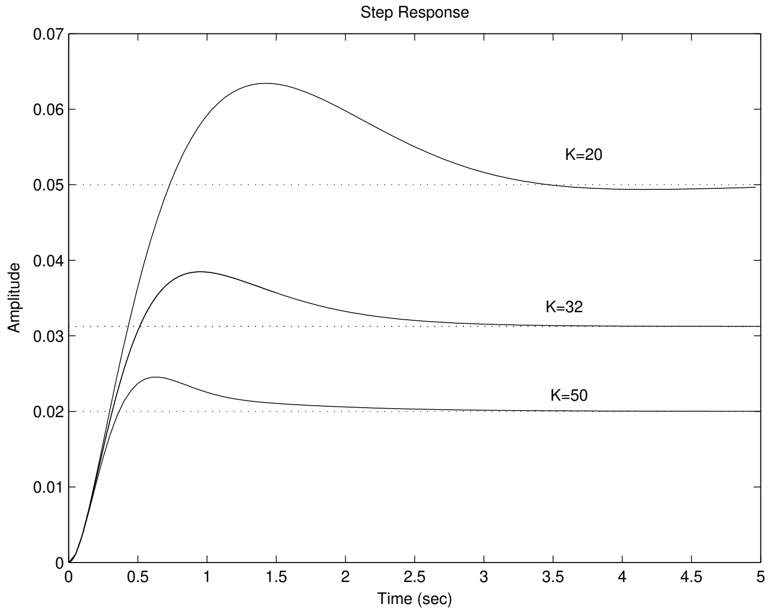

Figure 6. Step responses for $K = 20, 32, 50$ for problem 4.6.4:4.

8. Since

$$G(s) = \frac{s+1}{s^2(s+10)}$$

and $H(s) = K$,

$$\frac{C(s)}{R(s)} = \frac{s+1}{s^2(s+10)+K(s+1)}.$$

Thus, Figure 6 is produced by

```
>> K = 20;
>> num = [1 1];
>> den  = [1 10 K K];
>> step(num,den);
>> hold on
>> K = 32;
>> den  = [1 10 K K];
>> step(num,den,5);
>> K = 50;
>> den  = [1 10 K K];
>> step(num,den,5);
```

9. The following figures illustrate the block diagram algebra to reduce find the transfer function $\frac{Y(s)}{R(s)}$ for Problems 5 and 6.

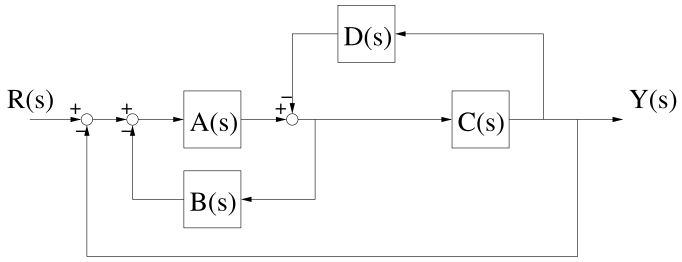

Figure 7. Block diagram for Problem 5.

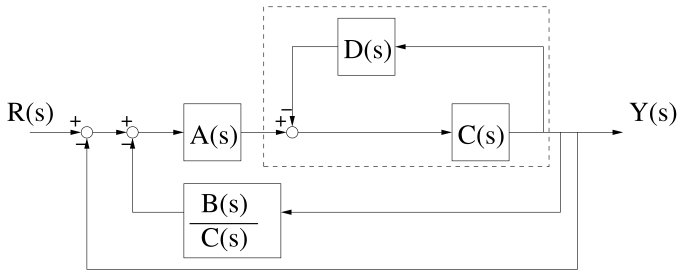

Figure 8. Block diagram for Problem 5.

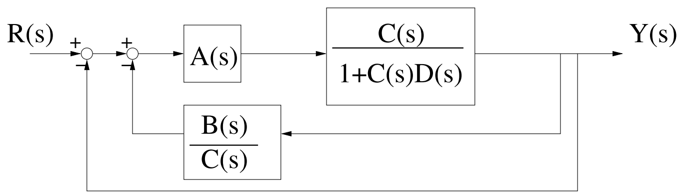

Figure 9. Block diagram for Problem 5.

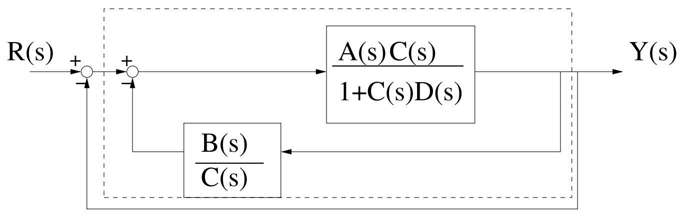

Figure 10. Block diagram for Problem 5.

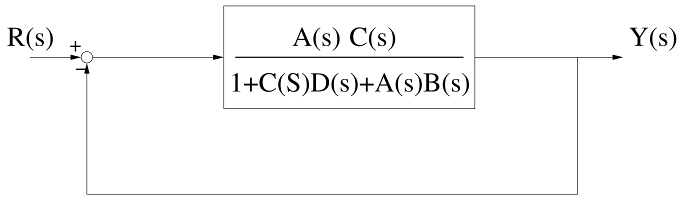

Figure 11. Block diagram for Problem 5.

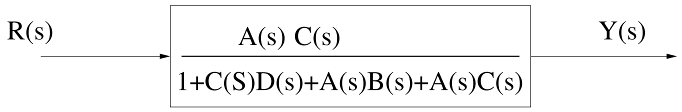

Figure 12. Block diagram for Problem 5.

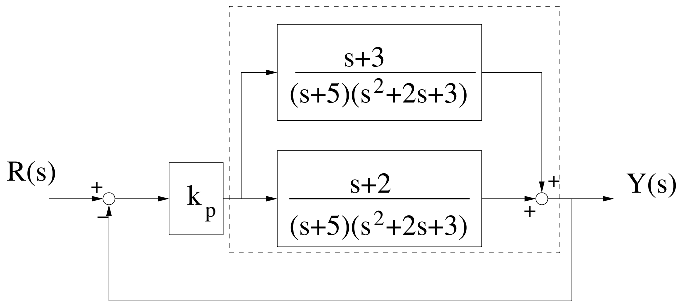

Figure 13. Block diagram for Problem 6.

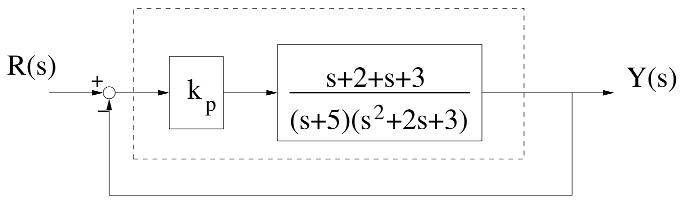

Figure 14. Block diagram for Problem 6.

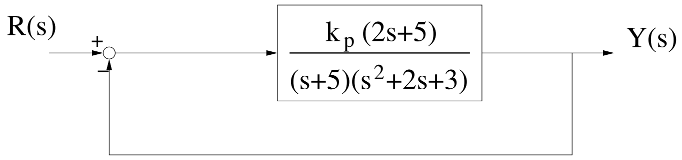

Figure 15. Block diagram for Problem 6.

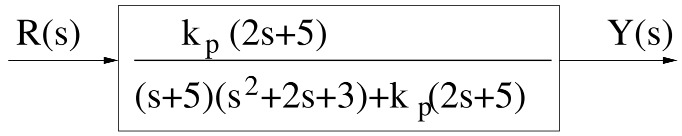

Figure 16. Block diagram for Problem 6.