University of Notre Dame
Aerospace and Mechanical Engineering

AME 437: Control Systems Engineering
Homework 8 Solutions

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1. (7.9.1:2) Typically, when lag compensation is implemented *after* compensation is designed for the transient response. Thus, we first find K such that $\zeta = \frac{1}{\sqrt{2}}$ and leaving out the lag compensator.

   Using Matlab,

   ```
   >> rlocus(1,[1 33 90])
   ```

   and resizing the axis limits by right clicking on the plot, produces Figure 1. Pointing and clicking indicates that $K = 46$ produces the desired damping ratio.

   For $K = 46$, the closed loop transfer function is

   $$\frac{C(s)}{R(s)} = \frac{460}{s^2 + 33s + 550}.$$

   Thus, for a step input

   $$c_{ss} = \lim_{s \to 0} s \frac{1}{s} \frac{460}{s^2 + 33s + 550} = 0.84.$$

   Reducing the steady state error by a factor of 4 will satisfy the steady state error specification. Therefore, pick $b = 0.4$.

   Using Matlab, and a little algebra

   ```
   >> step(460*[1 .4],[1 33.1 553.3 193])
   ```

   produces Figure 2, which illustrates that the steady state error is less than 5%.

2. (7.9.2:2) Consider

   $$G_c = k_p + \frac{k_i}{s} \qquad \text{and} \qquad G_p(s) = \frac{1}{(s+2)(s+4)}.$$

   (a) Note

   $$G_c = k_p \left(1 + \frac{0.1}{s} \right),$$

   so

   $$G_c(s)G_p(s) = \frac{k_p(1 + \frac{0.1}{s})}{(s+2)(s+4)} = \frac{k_p(s+0.1)}{s^3 + 6s^2 + 8s}.$$

   Using Matlab

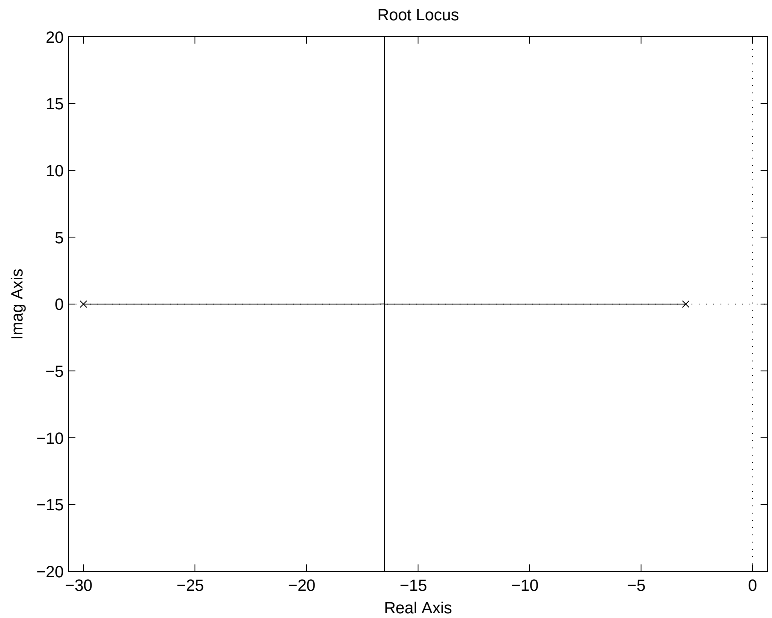

Figure 1. Root locus for Problem 1.

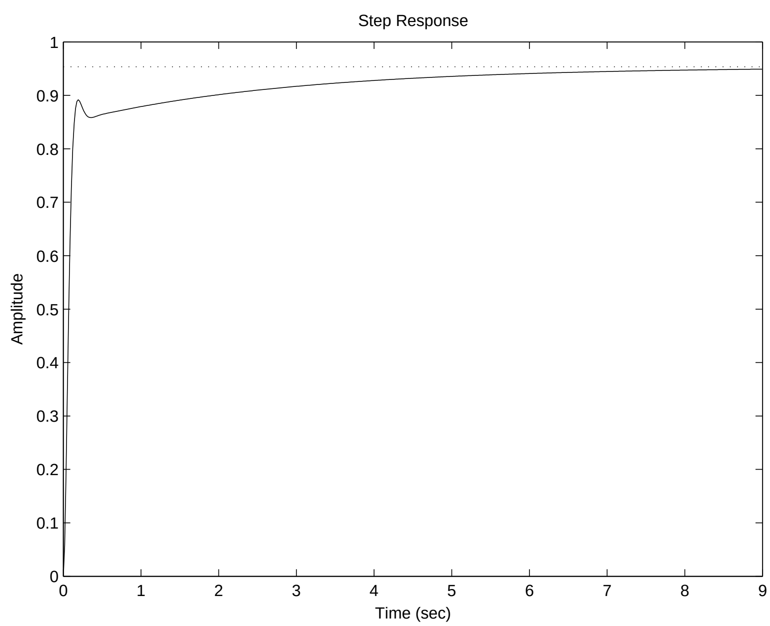

Figure 2. Step response for Problem 1.

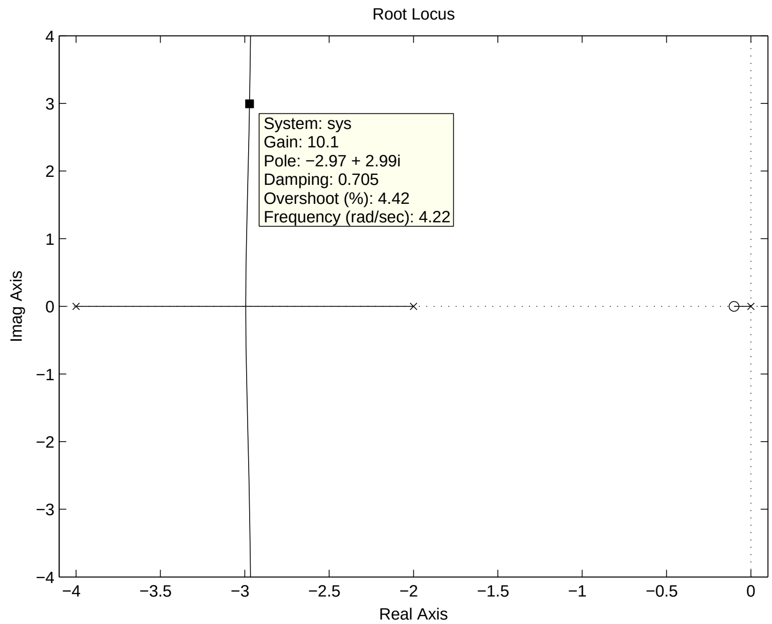

Figure 3. Root locus for Problem 2.

```
>> rlocus([1 0.1],[1 6 8 0])
```

produces Figure 3. Pointing and clicking gives that $k_p = 10$.

(b) The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{10s+1}{s^3+6s^2+18s+1}.$$

Clearly, $e_{ss} = 0$ for a step.

(c) Using octave

```
>> sys = tf2sys([10 1],[1 6 18 1])
>> step(sys)
```

produces figure 4.

3. Consider

$$G_p = \frac{10}{s(s+0.5)(s+20)} \qquad \text{and} \qquad G_c = \frac{K(s+b)}{s+10}.$$

(a) Using the angle criterion, illustrated in Figure 5, and using α for the angle from the zero located at $s = -b$ to $s = -s + 3i$

$$\alpha - (\tan^{-1}(3/17) + \tan^{-1}(3/7) + (180 - \tan^{-1}(3/2.5) + 135) = 180,$$

so

$$\alpha = 118.01°.$$

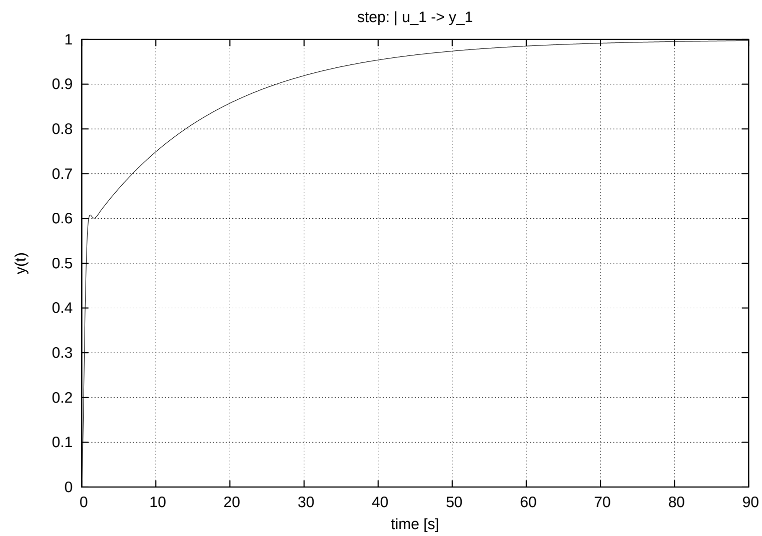

Figure 4. Step response for Problem 2.

Note: you need to keep track of the quadrant using $\tan^{-1}$. If you just punch this line into your calculator, it may not work. In matlab, I used the

```
atan2()
```

function.
Thus,

$$b = 1.404.$$

Using Matlab,

```
>> rlocus(10*[1 1.404],conv([1 10 0],[1 20.5 10]))
```

shows in Figure 6 that the root locus does go through the point $s = -3 + 3i$.

(b) From Figure 6, $K = 64$.

(c) From the definition of K_v,

$$K_v = \lim_{s \to 0} sG_c(s)G_p(s) = \lim_{s \to 0} \frac{64(s + 1.404)}{s + 10} \frac{10}{s(s + 0.5)(s + 20)} = 9.$$

4. (7.9.4:2) Consider

$$G_c(s) = k_p + k_d s \qquad \text{and} \qquad G_p(s) = \frac{1}{s(s + 0.5)}.$$

(a) To put the poles at $s = -3 \pm 2i$, first write

$$G_c(s) = k_d(s + b),$$

where $b = \frac{k_p}{k_d}$, and proceed as before.

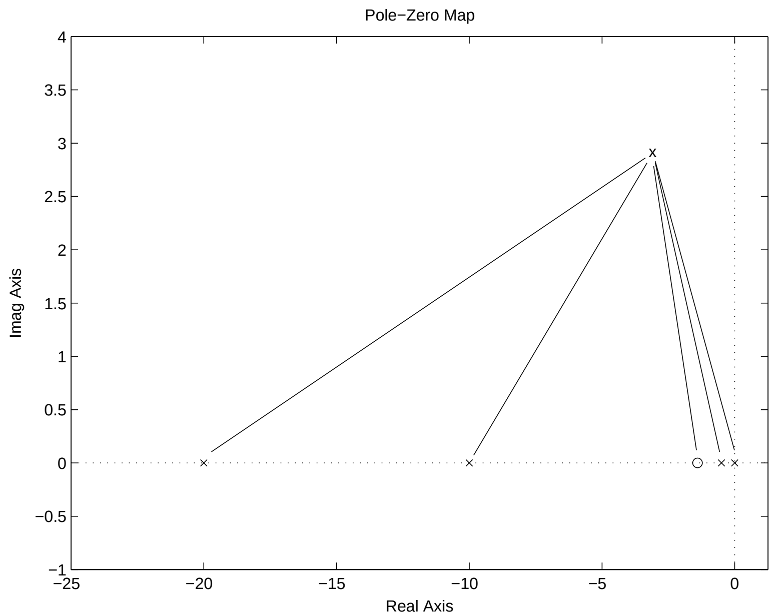

Figure 5. Angle criterion for problem 3.

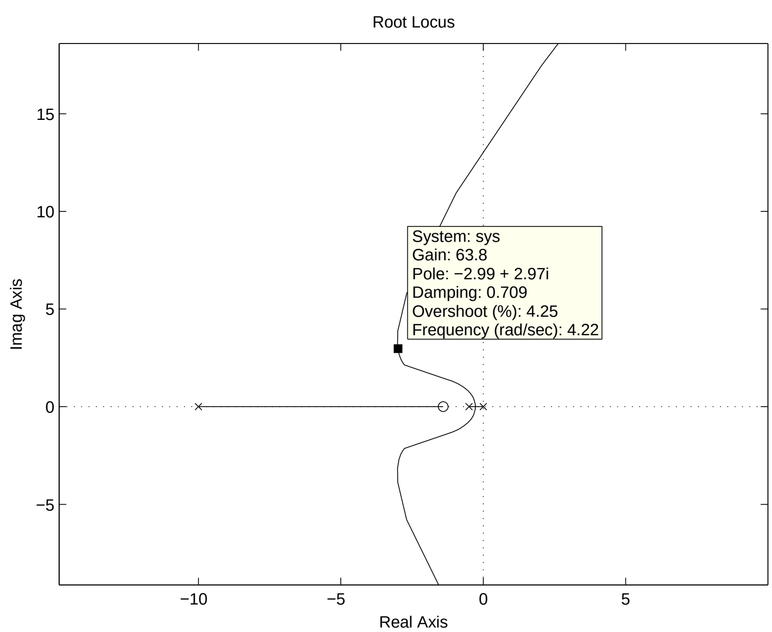

Figure 6. Root locus for problem 3.

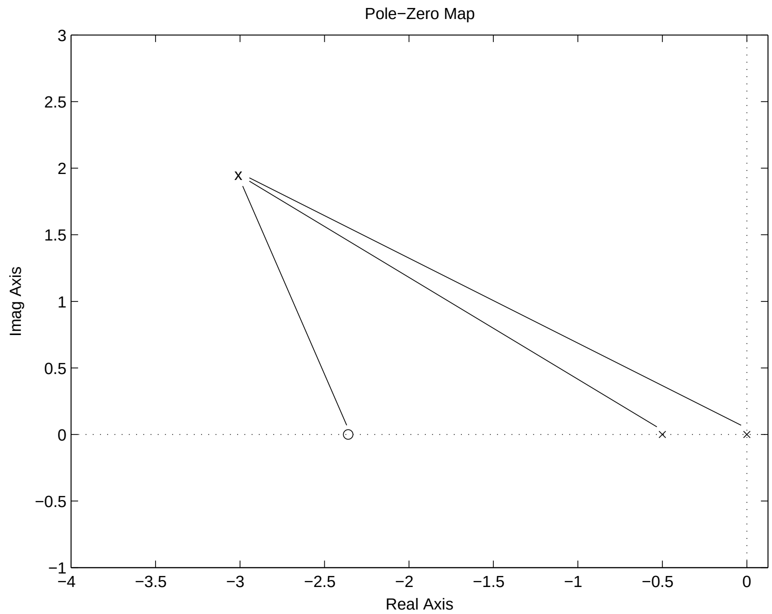

Figure 7. Angle criterion for problem 4.

Using the angle criterion again, as illustrated in Figure 7, and using α for the unknown angle from the zero

$$\alpha = 180 + \tan^{-1}(2/3) + \tan^{-1}(2/2.5) = 107.65.$$

Thus, $b = 2.36$.
Using Matlab,

```
>> rlocus([1 2.36],[1 .5 0])
```

as is illustrated in Figure 8, indicates the root locus goes through the point $s = -3 + 2i$. Also from the figure, $k_p = 5.4$.

(b) You can either directly compute the error, use the definition of K_v. This time, we will use K_v.

$$K_v = \lim_{s \to 0} s5.4(s + 2.36)\frac{1}{s(s + 0.5)} = 25.5.$$

Thus, for a ramp,

$$e_{ss} = \frac{1}{K_v} = 0.04.$$

(c) Skip c.

(d) Using Matlab

```
>> step(5.4*[1 2.36],[1  5.9 12.74])
```

is illustrated in Figure 9.

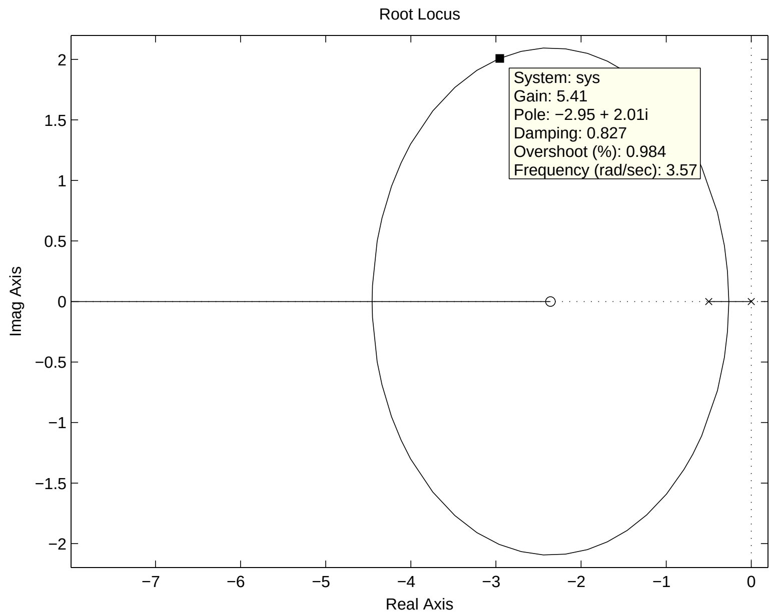

Figure 8. Root locus for problem 4.

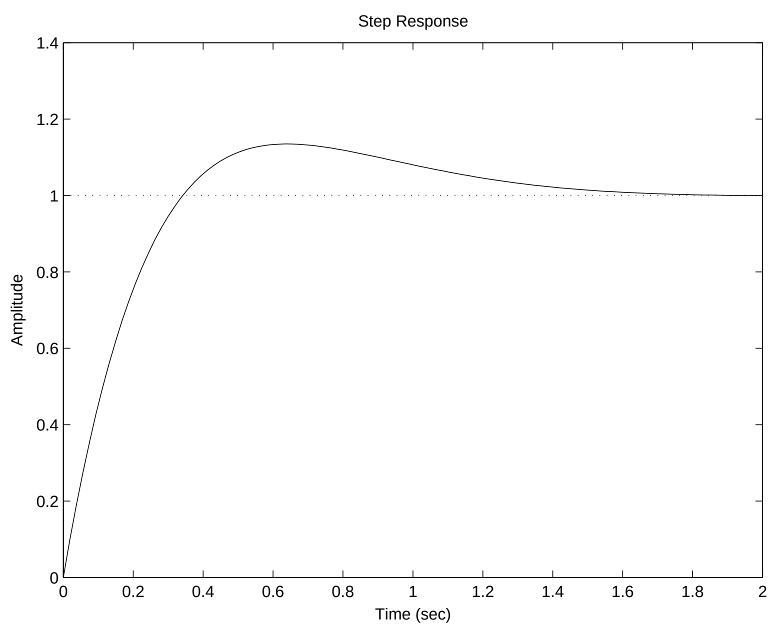

Figure 9. Step response for problem 4.

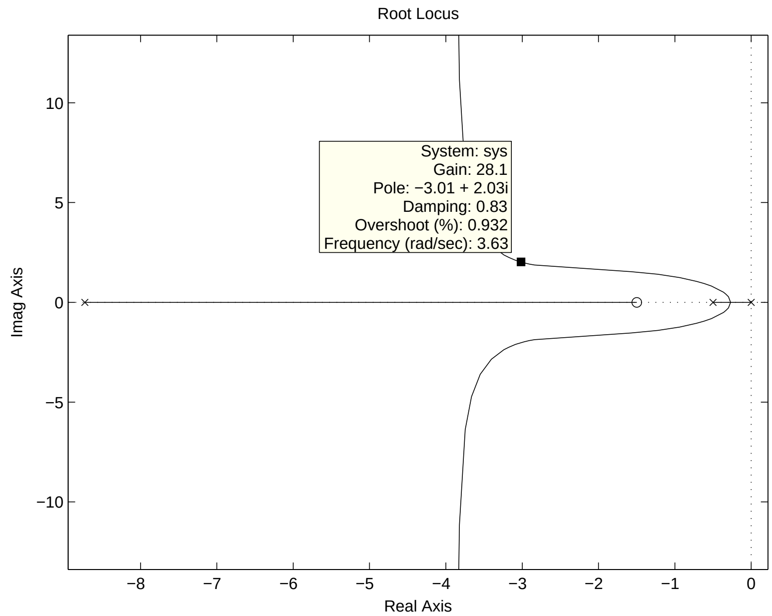

Figure 10. Root locus plot for problem 5.

5. (7.9.5:1) Consider

$$G_c(s) = \frac{K_c(s+1.5)}{s+p_1} \qquad \text{and} \qquad G_p(s) = \frac{1}{s(s+0.5)}.$$

(a) Using the angle criterion and denoting the unknown angle from p_1 by α,

$$\alpha = \tan^{-1}(2/1.5) - \tan^{-1}(2/2.5) - \tan^{-1}(2/3) - 180 = 19.2.$$

Thus,

$$p_1 = 8.73.$$

Using Matlab

```
>> rlocus([1 1.5],conv([1 8.73],[1 .5 0]))
```

indicates that $K_c = 28$, as is illustrated in Figure 10.

(b)

$$K_v = \lim_{s\to 0} s\frac{28(s+1.5)}{s+8.73}\frac{1}{s(s+0.5)} = 9.62,$$

so

$$e_{ss} = 0.105$$

for a ramp.

(c) The steady state error must be reduced by a factor of 6, so pick $z_1 = 0.1$ and $p_2 = 0.0167$.

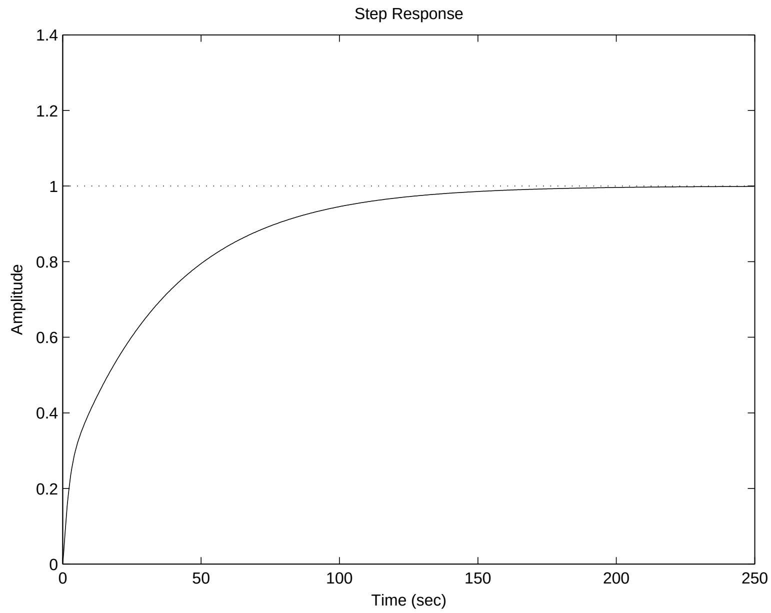

Figure 11. Step response for problem 5.

(d) The closed loop transfer function is

$$T_c(s) = \frac{(s+0.1)(s+1.5)}{(s+0.0167)(s+8.73)s(s+0.5)+(s+0.1)(s+1.5)}.$$

(e) Using Matlab,

```
>> step([1 1.6 0.15],[1 10.25 14.75 6.04 0.15])
```

gives the plot in Figure 11.

6. (7.9.6:2) Consider

$$G_c(s) = \frac{K(s+2)(s+z_2)}{s(s+10)} \qquad \text{and} \qquad G_p(s) = \frac{1}{(s+1)(s+2)}.$$

7. Using the angle criterion and α for the unknown angle from the zero to the point $s = -3+2i$,

$$\alpha = 180 + \tan^{-1}(2/7) + \tan^{-1}(2/2) + \tan^{-1}(2/3) = 117.3.$$

Thus,

$$z_2 = 1.97.$$

The root locus plot in Figure 12 illustrates that $K = 33$.

8.

$$K_v = \lim_{s\to 0} \frac{1}{s+1}\frac{33(s+2)}{s(s+10)} = 6.6.$$

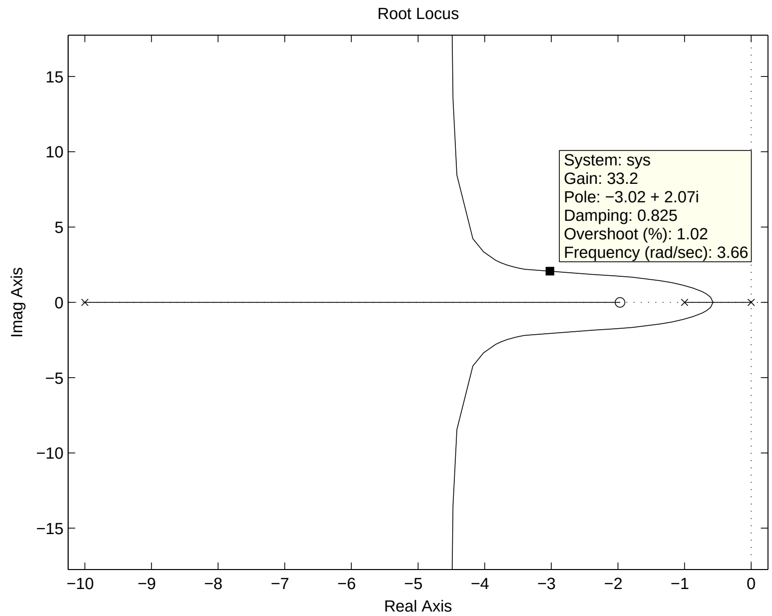

Figure 12. Root locus plot for problem 6.

Thus

$$e_{ss} = 0.15$$

for a ramp.

9. Skip c.
10. Using Matlab

```
>> step([33 66],[1 11 43 66])
```

is illustrated in Figure 13.

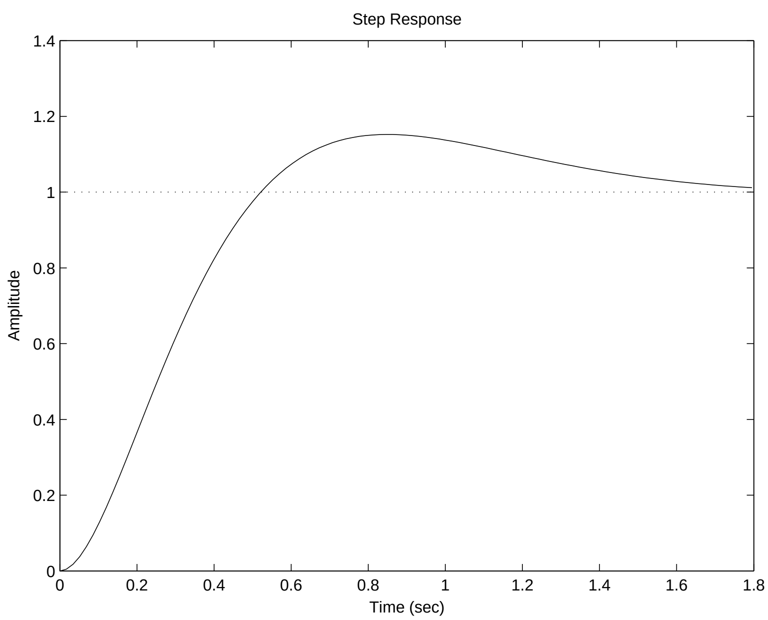

Figure 13. Step response for problem 6.