University of Notre Dame
Aerospace and Mechanical Engineering

ME 469: Introduction to Robotics

B. Goodwine
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I made a mistake in class today in response to a question. I was talking about how the order of the matrices representing rotations was important, so that

$$R_x R_y R_z \neq R_z R_y R_x.$$

This is absolutely correct, and the way to think about it was to think of how ${}^A_B R$ "acted" on a vector, say ${}^B P$:

$${}^A P = R_x R_y R_z \ {}^B P.$$

For the fixed axis case, if rotation about the z–axis was the first rotation (remember that the order of rotations is *specified*), the R_z has to be next to ${}^B P$ to act on it first. *Then*, R_y acts on it, etc. So far, everything is correct. Then someone asked if we put ${}^B P$ on the *left* instead of the right, if

$${}^A_B R = {}^B P R_z R_y R_x$$

(reversing the order) would be correct. I said, yes, except you would have to write ${}^B P$ as ${}^B P^T$ (the transpose) to make the multiplication work out correctly.
Recall that

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^T = [x_1 \quad x_2 \quad x_3].$$

In other words, what I implied, (which is wrong) it that

$${}^A P = R_x R_y R_z \ {}^B P = \ {}^B P^T R_z R_y R_x.$$

The right answer is that R_z, R_y, and R_x would have to be transposed as well. The right answer is

$$\left({}^A_B R \quad {}^B P\right)^T = \left(R_x R_y R_z \ {}^B P\right)^T = \ {}^B P^T R_z^T R_y^T R_x^T.$$

This is a consequence of the fact that for matrices,

$$(AB)^T = B^T A^T.$$