I made a mistake in class today in response to a question. I was talking about how the order of the matrices representing rotations was important, so that
\[ R_x R_y R_z \neq R_z R_y R_x. \]
This is absolutely correct, and the way to think about it was to think of how \( A_B R \) “acted” on a vector, say \( B P \):
\[ A_P = R_x R_y R_z B P. \]
For the fixed axis case, if rotation about the \( z \)-axis was the first rotation (remember that the order of rotations is specified), the \( R_z \) has to be next to \( B P \) to act on it first. Then, \( R_y \) acts on it, etc. So far, everything is correct. Then someone asked if we put \( B P \) on the left instead of the right, if
\[ A_B R = B P R_z R_y R_x \]
(reversing the order) would be correct. I said, yes, except you would have to write \( B P \) as \( B P^T \) (the transpose) to make the multiplication work out correctly.
Recall that
\[ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^T = [x_1 \ x_2 \ x_3]. \]
In other words, what I implied, (which is wrong) it that
\[ A_P = R_x R_y R_z B P = B P^T R_z R_y R_x. \]
The right answer is that \( R_z, R_y, \) and \( R_x \) would have to be transposed as well. The right answer is
\[ (A_B R \ B P)^T = (R_x R_y R_z \ B P)^T = B P^T R_z^T R_y^T R_x^T. \]
This is a consequence of the fact that for matrices,
\[ (AB)^T = B^T A^T. \]