In[1]:=
```
<< forward.m
```

In[2]:=
```
?? T
```

```
Global`T

T[alph_, a_, d_, th_] := {{Cos[th], -Sin[th], 0, a},
  {Sin[th] * Cos[alph], Cos[th] * Cos[alph], -Sin[alph], -(Sin[alph] * d)},
  {Sin[th] * Sin[alph], Cos[th] * Sin[alph], Cos[alph], Cos[alph] * d}, {0, 0, 0, 1}}
```

In[3]:=
```
t = T[0, 0, 0, θ1] . T[0, l1, h1, θ2] . T[0, l2, 0, θ3] . T[0, 0, d4, 0] // Simplify
```

Out[3]=
$$\{\{\text{Cos}[\theta_1+\theta_2+\theta_3], -\text{Sin}[\theta_1+\theta_2+\theta_3], 0, \text{Cos}[\theta_1]\, l_1+\text{Cos}[\theta_1+\theta_2]\, l_2\},\ \{\text{Sin}[\theta_1+\theta_2+\theta_3], \text{Cos}[\theta_1+\theta_2+\theta_3], 0, \text{Sin}[\theta_1]\, l_1+\text{Sin}[\theta_1+\theta_2]\, l_2\},\ \{0, 0, 1, d_4+h_1\},\ \{0, 0, 0, 1\}\}$$

```
t
```

$$\{\{\text{Cos}[\theta_1+\theta_2+\theta_3], -\text{Sin}[\theta_1+\theta_2+\theta_3], 0, \text{Cos}[\theta_1]\, l_1+\text{Cos}[\theta_1+\theta_2]\, l_2\},\ \{\text{Sin}[\theta_1+\theta_2+\theta_3], \text{Cos}[\theta_1+\theta_2+\theta_3], 0, \text{Sin}[\theta_1]\, l_1+\text{Sin}[\theta_1+\theta_2]\, l_2\},\ \{0, 0, 1, d_4+h_1\},\ \{0, 0, 0, 1\}\}$$

In[4]:=
```
% // MatrixForm
```

Out[4]//MatrixForm=
$$\begin{pmatrix} \text{Cos}[\theta_1+\theta_2+\theta_3] & -\text{Sin}[\theta_1+\theta_2+\theta_3] & 0 & \text{Cos}[\theta_1]\, l_1+\text{Cos}[\theta_1+\theta_2]\, l_2 \\ \text{Sin}[\theta_1+\theta_2+\theta_3] & \text{Cos}[\theta_1+\theta_2+\theta_3] & 0 & \text{Sin}[\theta_1]\, l_1+\text{Sin}[\theta_1+\theta_2]\, l_2 \\ 0 & 0 & 1 & d_4+h_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[5]:=
```
p = Part[Transpose[t], 4]
```

Out[5]=
$$\{\text{Cos}[\theta_1]\, l_1+\text{Cos}[\theta_1+\theta_2]\, l_2,\ \text{Sin}[\theta_1]\, l_1+\text{Sin}[\theta_1+\theta_2]\, l_2,\ d_4+h_1,\ 1\}$$

In[6]:=
```
p // MatrixForm
```

Out[6]//MatrixForm=
$$\begin{pmatrix} \text{Cos}[\theta_1]\, l_1+\text{Cos}[\theta_1+\theta_2]\, l_2 \\ \text{Sin}[\theta_1]\, l_1+\text{Sin}[\theta_1+\theta_2]\, l_2 \\ d_4+h_1 \\ 1 \end{pmatrix}$$

In[7]:=
```
v = D[p, θ1] t1d + D[p, θ2] t2d + D[p, θ3] t3d + D[p, d4] d4d // Simplify
```

Out[7]=
$$\{-t1d\, \text{Sin}[\theta_1]\, l_1-(t1d+t2d)\, \text{Sin}[\theta_1+\theta_2]\, l_2,\ t1d\, \text{Cos}[\theta_1]\, l_1+(t1d+t2d)\, \text{Cos}[\theta_1+\theta_2]\, l_2,\ d4d,\ 0\}$$

In[8]:=
```
v // MatrixForm
```

Out[8]//MatrixForm=
$$\begin{pmatrix} -t1d\, \text{Sin}[\theta_1]\, l_1-(t1d+t2d)\, \text{Sin}[\theta_1+\theta_2]\, l_2 \\ t1d\, \text{Cos}[\theta_1]\, l_1+(t1d+t2d)\, \text{Cos}[\theta_1+\theta_2]\, l_2 \\ d4d \\ 0 \end{pmatrix}$$

In[17]:=
```
R = Transpose[Drop[Transpose[Drop[t, -1]], -1]]
```

Out[17]=
$$\{\{\text{Cos}[\theta_1+\theta_2+\theta_3], -\text{Sin}[\theta_1+\theta_2+\theta_3], 0\}, \{\text{Sin}[\theta_1+\theta_2+\theta_3], \text{Cos}[\theta_1+\theta_2+\theta_3], 0\}, \{0, 0, 1\}\}$$

In[18]:= **R // MatrixForm**

Out[18]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta_1+\theta_2+\theta_3] & -\text{Sin}[\theta_1+\theta_2+\theta_3] & 0 \\ \text{Sin}[\theta_1+\theta_2+\theta_3] & \text{Cos}[\theta_1+\theta_2+\theta_3] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[11]:= **Rdot = D[R, θ_1] t1d + D[R, θ_2] t2d + D[R, θ_3] t3d + D[R, d_4] d4d // Simplify**

```
General::spell1 :
 Possible spelling error: new symbol name "Rdot" is similar to existing symbol "Root".
```

Out[11]= $\{\{-(t1d+t2d+t3d)\,\text{Sin}[\theta_1+\theta_2+\theta_3],\ (t1d+t2d+t3d)\,\text{Cos}[\theta_1+\theta_2+\theta_3],\ 0\},\ \{-(t1d+t2d+t3d)\,\text{Cos}[\theta_1+\theta_2+\theta_3],\ -(t1d+t2d+t3d)\,\text{Sin}[\theta_1+\theta_2+\theta_3],\ 0\},\ \{0,\ 0,\ 0\}\}$

In[12]:= **omega = Rdot . Transpose[R] // Simplify**

Out[12]= $\{\{0,\ t1d+t2d+t3d,\ 0\},\ \{-t1d-t2d-t3d,\ 0,\ 0\},\ \{0,\ 0,\ 0\}\}$

In[13]:= **omega // MatrixForm**

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 & t1d+t2d+t3d & 0 \\ -t1d-t2d-t3d & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[15]:= **V = Flatten[{Drop[v, -1], {0, 0, -t1d - t2d - t3d}}]**

Out[15]= $\{-t1d\,\text{Sin}[\theta_1]\,l_1-(t1d+t2d)\,\text{Sin}[\theta_1+\theta_2]\,l_2,\ t1d\,\text{Cos}[\theta_1]\,l_1+(t1d+t2d)\,\text{Cos}[\theta_1+\theta_2]\,l_2,\ d4d,\ 0,\ 0,\ -t1d-t2d-t3d\}$

In[16]:= **V // MatrixForm**

Out[16]//MatrixForm=

$$\begin{pmatrix} -t1d\,\text{Sin}[\theta_1]\,l_1-(t1d+t2d)\,\text{Sin}[\theta_1+\theta_2]\,l_2 \\ t1d\,\text{Cos}[\theta_1]\,l_1+(t1d+t2d)\,\text{Cos}[\theta_1+\theta_2]\,l_2 \\ d4d \\ 0 \\ 0 \\ -t1d-t2d-t3d \end{pmatrix}$$