1. (Craig, 2.3)

The answer for this can be found in the Appendix. In the notation of the book, $^A_B R_{Z'X'Y'}(\alpha, \beta, \gamma)$ stands for the $ZXY$ Euler angle matrix. We can look this up in the Appendix, and see that if $\alpha = \theta$, $\beta = \phi$ and $\gamma = 0$, then

$$
^A_B R_{Z'X'Y'}(\theta, \phi, 0) = \begin{bmatrix}
\cos \theta & -\sin \theta \cos \phi & \sin \theta \sin \phi \\
\sin \theta & \cos \theta \cos \phi & -\cos \theta \sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
$$

You could also used the formula for $R_{Z'X'Z'}$ to get the same answer since the third rotation was not specified.

2. (Craig, 2.5)

The definition of an eigenvector is any vector $\phi$ that satisfies the equation

$$
^A_B R \phi = \lambda \phi
$$

for some $\lambda$. If $\lambda = 1$, then the equation is

$$
^A_B R \phi = \phi
$$

which indicates that $\phi$ is not rotated by $^A_B R$. The only vector that is not changed by a rotation is the axis of rotation.

3. (Craig, 2.6)

????????? Hopefully, someone will get this problem right.

4. (Craig, 2.14)

We are looking for $^A_B T$, where, starting from initial coincidence, frame $B$ is rotated by $\theta$ about $K$, where $K$ (a unit vector, not a matrix) passes through the point $^A P$.

To do this, we will define two other frames, $A'$ and $B'$, where the origins of $A'$ and $B'$ are located at the point $^A P$, $A$ and $A'$ have the same orientation, $B'$ is rotated relative to $A'$ by an amount $\theta$ about $\hat{K}$, and $B'$ and $B$ have the same orientation. See Figure 2.20 in the book. This idea is exactly the same idea as Example 2.9 in the book.
Now, we know that
\[ \frac{A}{B}T = \frac{A}{A'} T_{B'} B' T. \]

Since \( A \) and \( A' \) only differ by a translation,
\[ \frac{A}{A'} T = \begin{bmatrix} I & A' \end{bmatrix} \]
where \( I \) is the \( 3 \times 3 \) identity matrix.

Now, \( A' \) and \( B' \) only differ by a rotation about \( \hat{K} \). Therefore,
\[ \frac{A'}{B'} T = \begin{bmatrix} R_K(\theta) & 0 \\ 0 & 1 \end{bmatrix}, \]
where \( R_K(\theta) \) is given by Rodrigues’ formula (like I like) or Equation 2.80 in the book.

Finally, \( B' \) and \( B \) differ only by translation along negative \( A'P \), i.e.,
\[ \frac{B'}{B} T = \begin{bmatrix} I & -A' \end{bmatrix} \]

Multiplying everything together gives:
\[
\frac{A}{B}T = \begin{bmatrix} I & A' \end{bmatrix} \begin{bmatrix} R_K(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & -A' \end{bmatrix} = \begin{bmatrix} R_K(\theta) - R_K(\theta)A' + A' \end{bmatrix}.
\]

5. (Craig, 2.16)
Covered in class on Wednesday, September 16, 1998.

6. (Craig, 2.20)
Let
\[ Q = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}. \]

Then,
\[ Q' = R_K(\theta)Q = \begin{bmatrix} k_x k_x q_x v\theta + q_x c\theta + k_x k_y q_y v\theta - k_x q_y s\theta + k_x k_z q_z v\theta + k_y q_z s\theta \\ k_x k_y q_y v\theta + k_y k_z q_z v\theta + q_y c\theta + k_y k_x q_x v\theta + k_z q_x s\theta \\ k_y k_z q_z v\theta - k_y q_z s\theta + k_y k_z q_z v\theta + k_z q_y s\theta + k_z k_z q_z v\theta + q_z c\theta \end{bmatrix}. \]

Note that each row has a term of the form \( q_i c\theta \). Separating these terms will give the \( Q \cos \theta \) term.

Note also that
\[ (1 - \cos \theta)(K \cdot Q)K = \begin{bmatrix} v(k_x q_x + k_y q_y + k_z q_z)k_x \\ v(k_x q_x + k_y q_y + k_z q_z)k_y \\ v(k_x q_x + k_y q_y + k_z q_z)k_z \end{bmatrix}. \]
where, as usual, $v = 1 - \cos \theta$.

Finally, note that
\[ \sin \theta (K \times Q) = \begin{bmatrix} \sin \theta(-k_z q_y + k_y q_z) \\ \sin \theta(k_z q_x - k_x q_z) \\ \sin \theta(k_y q_x - k_x q_y) \end{bmatrix}. \]

Inspecting Equation 1 verifies that
\[ Q' = Q \cos \theta + \sin \theta (K \times Q) + (1 - \cos \theta)(K \cdot Q) K, \]
as required.

7. (Craig, 2.38)
For two unit vectors, we know that the cosine of the angle between them is given by their dot product.

Since,
\[ v_1 \cdot v_2 = v_{1x} v_{2x} + v_{1y} v_{2y} + v_{1z} v_{2z}, \]
where
\[ v_1 = \begin{bmatrix} v_{1x} \\ v_{1y} \\ v_{1z} \end{bmatrix} \quad \text{and} \quad v_2 = \begin{bmatrix} v_{2x} \\ v_{2y} \\ v_{2z} \end{bmatrix}, \]
these dot products can be written in vector form
\[ v_1 \cdot v_2 = v_1^T v_2 = \begin{bmatrix} v_{1x} & v_{1y} & v_{1z} \end{bmatrix} \begin{bmatrix} v_{2x} \\ v_{2y} \\ v_{2z} \end{bmatrix}. \]

Let $R$ be a rotation matrix. If the angle between vectors is preserved by a rigid body rotation, the cosine of the angle is preserved. Therefore,
\[ v_1^T v_2 = (R v_1)^T (R v_2) = v_1^T R^T R v_2. \]

Since this is true for any two vectors $v_1$ and $v_2$, it must be the case that
\[ R^T R = I, \]
which shows that the transpose of a rotation matrix is its inverse.