University of Notre Dame
Aerospace and Mechanical Engineering

ME 469: Introduction to Robotics
Homework 2 Solutions

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1. This is simply Equation 3.6 implemented in Mathematica:

```
T[alph_, a_, d_, th_] :=
  {{Cos[th], -Sin[th], 0, a},
    {Sin[th]*Cos[alph], Cos[th]*Cos[alph], -Sin[alph], -Sin[alph]*d},
    {Sin[th]*Sin[alph], Cos[th]*Sin[alph], Cos[alph], Cos[alph]*d},
    {0, 0, 0, 1}}
```

   Can be used, for example, as

```
T[Pi/2,0,0,theta] //MatrixForm
```

   which will print out transformation with those parameter values in a nice matrix form in the Mathematica notebook.

2. (Craig, 3.4)
   Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 1.

   Consulting the figure, the following link parameters are apparent:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\frac{\pi}{2}$	0	0	θ_2
3	0	L_3	0	θ_3

   Determining ${}^0_1T$,${}^1_2T$ and ${}^2_3T$ is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.

$$
{}^0_1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

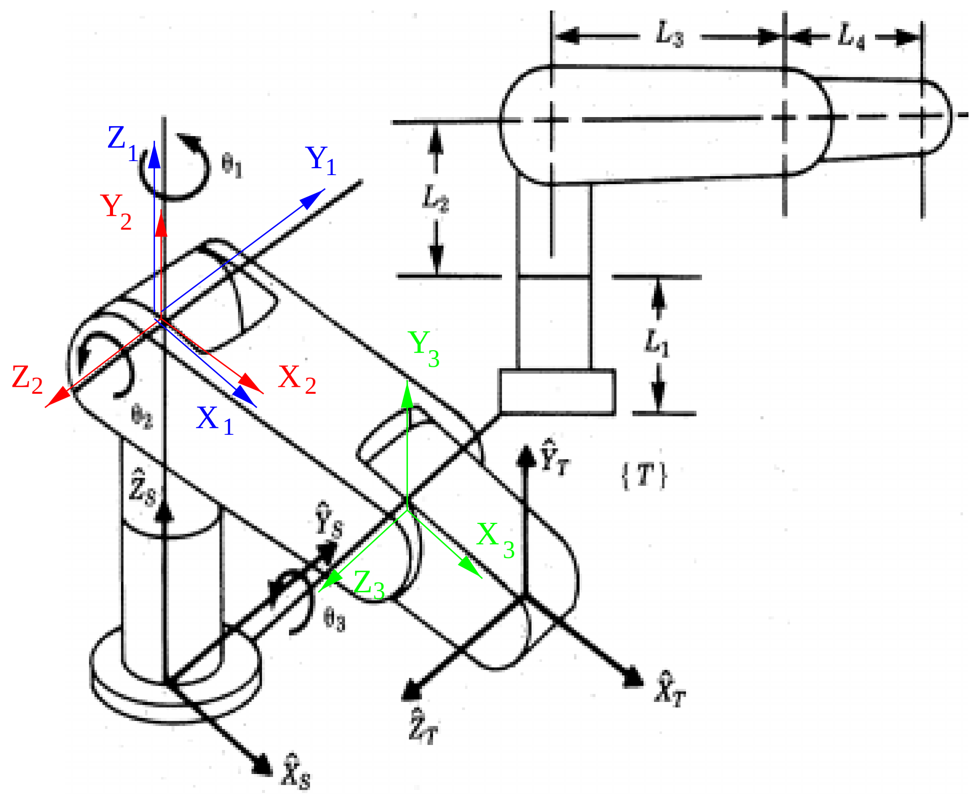

Figure 1. Frames for problem 2.

$$
{}^1_2T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

3. (Craig, 3.9)
   We were given ${}^0_2T$, and want to find the location of the tip in fram 0.

   Clearly,

   $$ {}^0P_{\text{tip}} = {}^0_2T \quad {}^2P_{\text{tip}}, $$

   where

   $$ {}^2P_{\text{tip}} = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}, $$

   or

   $$ \boxed{ {}^0P_{\text{tip}} = \begin{bmatrix} l_2\cos\theta_1\cos\theta_2 + l_1\cos\theta_1 \\ l_2\sin\theta_1\cos\theta_2 + l_1\sin\theta_1 \\ l_2\sin\theta_2 \end{bmatrix}. } $$

4. (Craig, 3.11)
   Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 2.

   Consulting the figure, the following link parameters are apparent:

i	α_{i-1}	a_{i-1}	d_i	θ_i
4	0	0	0	θ_4
5	ϕ	0	0	θ_5
6	$-\phi$	0	0	θ_6

   Determining ${}^B_4T$,${}^4_5T$ and ${}^5_6T$ is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.

```
<</home/bill/courses/me469/math/forward.m
T[0,0,0,t4] . T[phi,0,0,t5] . T[-phi,0,0,t6] //Simplify
```

   produces the mess

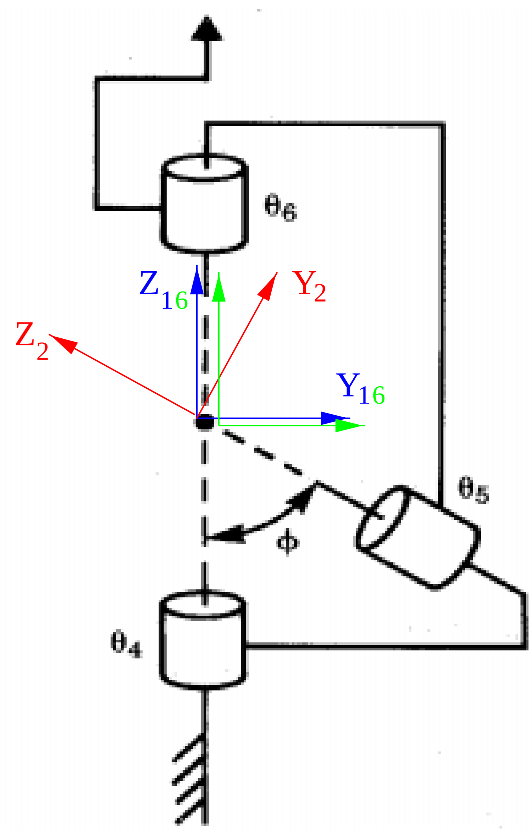

Figure 2. Frames for problem 4.

```
{-(Sin[t4]*(Cos[phi]*Cos[t6]*Sin[t5] +
         Cos[phi]^2*Cos[t5]*Sin[t6] + Sin[phi]^2*Sin[t6])) +
    Cos[t4]*(Cos[t5]*Cos[t6] - Cos[phi]*Sin[t5]*Sin[t6]),
   -(Cos[phi]^2*Cos[t5]*Cos[t6]*Sin[t4]) -
    Cos[t6]*Sin[phi]^2*Sin[t4] -
    Cos[phi]*Cos[t4 + t6]*Sin[t5] - Cos[t4]*Cos[t5]*Sin[t6],
   -(Sin[phi]*(Cos[phi]*(-1 + Cos[t5])*Sin[t4] +
        Cos[t4]*Sin[t5])), 0},
  {Cos[phi]*Cos[t4 + t6]*Sin[t5] +
    Cos[t4]*Sin[phi]^2*Sin[t6] +
    Cos[t5]*(Cos[t6]*Sin[t4] + Cos[phi]^2*Cos[t4]*Sin[t6]),
   Cos[phi]^2*Cos[t4]*Cos[t5]*Cos[t6] +
    Cos[t4]*Cos[t6]*Sin[phi]^2 - Cos[t5]*Sin[t4]*Sin[t6] -
    Cos[phi]*Sin[t5]*Sin[t4 + t6],
   Sin[phi]*(Cos[phi]*Cos[t4]*(-1 + Cos[t5]) -
      Sin[t4]*Sin[t5]), 0},
  {Sin[phi]*(Cos[t6]*Sin[t5] +
      Cos[phi]*(-1 + Cos[t5])*Sin[t6]),
   Sin[phi]*(Cos[phi]*(-1 + Cos[t5])*Cos[t6] -
      Sin[t5]*Sin[t6]), Cos[phi]^2 + Cos[t5]*Sin[phi]^2, 0},
  {0, 0, 0, 1}}
```

To check to make sure it makes sense, take all the joint angles, θ_4, θ_5 and θ_6 to be zero:

```
<</home/bill/courses/me469/math/forward.m
T[0,0,0,0] . T[phi,0,0,0] . T[-phi,0,0,0] //Simplify //MatrixForm
```

which gives the expected answer

$$ {}^B_6T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

If $\theta_4 = \pi$ and the other joints are zero, then

```
<</home/bill/courses/me469/math/forward.m
T[0,0,0,Pi] . T[phi,0,0,0] . T[-phi,0,0,0] //Simplify //MatrixForm
```

$$ {}^B_6T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

which also is expected because rotating joint 4 by π will rotate the x and y components of a vector by 180°, which will make their components negative of what they start as.

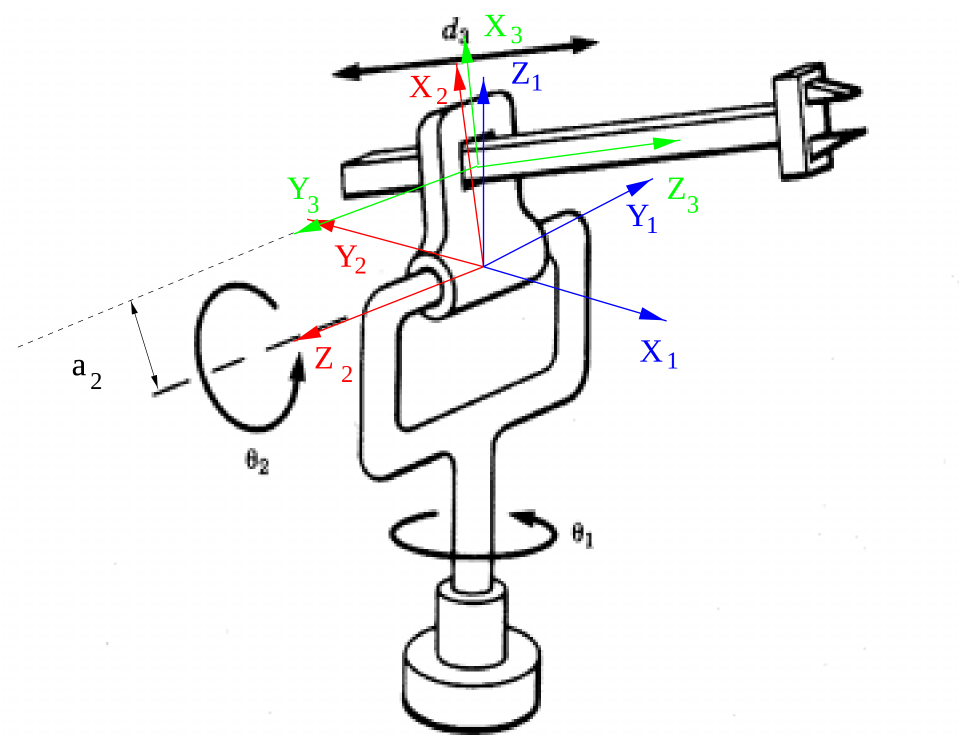

Figure 3. Frames for problem 5.

5. (Craig, 3.17)
Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 3.

Consulting the figure, and naming the distance a_2, the following link parameters are apparent:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\frac{\pi}{2}$	0	0	$\theta_2 + \frac{\pi}{2}$
3	$\frac{\pi}{2}$	a_2	d_3	0

bf: Note the $\frac{\pi}{2}$ term added to θ_2!

Determining ${}^0_1T$,${}^1_2T$ and ${}^2_3T$ is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.

```
<</home/bill/courses/me469/math/forward.m
T[0,0,0,t1] . T[Pi/2,0,0,t2+Pi/2] . T[Pi/2,a2,d3,0] //Simplify
```

produces the mess

```
{{-Cos[t1]*Sin[t2], Sin[t1], Cos[t1]*Cos[t2],
   Cos[t1]*(d3*Cos[t2] - a2*Sin[t2])},
  {-Sin[t1]*Sin[t2], -Cos[t1], Cos[t2]*Sin[t1],
   Sin[t1]*(d3*Cos[t2] - a2*Sin[t2])},
  {Cos[t2], 0, Sin[t2], a2*Cos[t2] + d3*Sin[t2]},
  {0, 0, 0, 1}}
```

or

$$ {}^{3}_{0}T = \begin{bmatrix} -\cos\theta_1 \sin\theta_2 & \sin\theta_1 & \cos\theta_1 \cos\theta_2 & \cos\theta_1 (d_3 \cos\theta_2 - a_2 \sin\theta_2) \\ -\sin\theta_1 \sin\theta_2 & -\cos\theta_1 & \cos\theta_2 \sin\theta_1 & \sin\theta_1 (d_3 \cos\theta_2 - a_2 \sin\theta_2 a_2) \\ \cos\theta_2 & 0 & \sin\theta_2 & a_2 \cos\theta_2 + d_3 \sin\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

To check to make sure it makes sense, take all the joint angles, θ_1, θ_2 and d_3 to be zero:

```
<</home/bill/courses/me469/math/forward.m
T[0,0,0,0] . T[Pi/2,0,0,0] . T[Pi/2,a2,0,0] //Simplify
```

which gives the expected answer

$$ {}^{0}_{3}T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

6. (Craig, 3.18)
Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 4.

Consulting the figure, and naming the distance a_2, the following link parameters are apparent:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\frac{\pi}{2}$	0	0	$\theta_2 + \frac{\pi}{2}$
3	0	a_2	0	θ_3

bf: Note the $\frac{\pi}{2}$ term added to θ_2!

Determining ${}^{0}_{1}T$,${}^{1}_{2}T$ and ${}^{2}_{3}T$ is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.

```
<</home/bill/courses/me469/math/forward.m
T[0,0,0,t1] . T[Pi/2,0,0,t2+Pi/2] . T[Pi/2,a2,d3,0] //Simplify
```

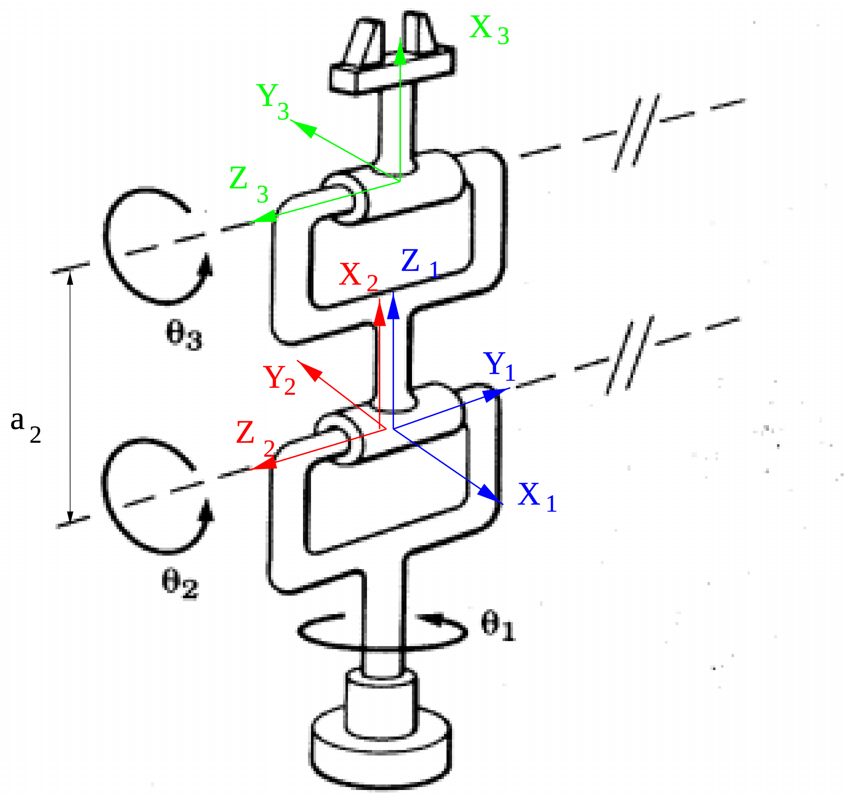

Figure 4. Frames for problem 6.

gives

$$
{}^3_0T = \begin{bmatrix} -\cos\theta_1\sin(\theta_2+\theta_3) & -\cos\theta_1\cos(\theta_2+\theta_3) & \sin\theta_1 & -a_2\cos\theta_1\sin\theta_2 \\ -\sin\theta_1\sin(\theta_2+\theta_3) & -\cos(\theta_2+\theta_3)\,\sin\theta_1 & -\cos\theta_1 & -a_2\sin\theta_1\sin\theta_2 \\ \cos(\theta_2+\theta_3) & -\sin(\theta_2+\theta_3) & 0 & a_2\cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

To check to make sure it makes sense, take all the joint angles, θ_1, θ_2 and θ_3 to be zero:

```
<</home/bill/courses/me469/math/forward.m
T[0,0,0,0] . T[Pi/2,0,0,Pi/2] . T[0,a2,0,0] //Simplify
```

which gives the expected answer

$$
{}^0_3T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

7. (Craig, 3.20)
   Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 5.

   Consulting the figure, and naming the distances a_1 and a_2, the following link parameters are apparent:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	0
2	0	a_1	0	θ_2
3	0	a_2	0	θ_3

   Determining ${}^0_1T$,${}^1_2T$ and ${}^2_3T$ is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.

```
<</home/bill/courses/me469/math/forward.m
T[0,0,d1,0] . T[0,a1,0,t2] . T[0,a2,0,t3] //Simplify
```

   gives

$$
{}^3_0T = \begin{bmatrix} \cos(\theta_2+\theta_3) & -\sin(\theta_2+\theta_3) & 0 & a_1+\cos(\theta_2)\,a_2 \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & \sin(\theta_2)\,a_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

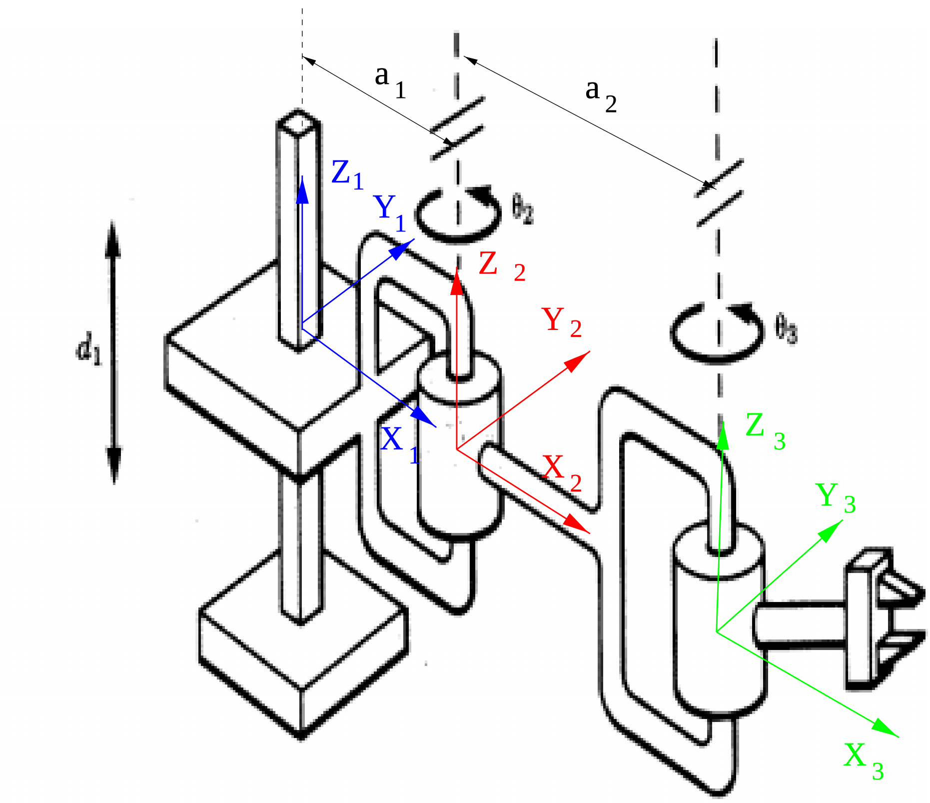

Figure 5. Frames for problem 7.