1. This is simply Equation 3.6 implemented in Mathematica:

\[
T[\alpha, a, d, \theta] :=
\begin{bmatrix}
\cos \theta & -\sin \theta & 0 & a \\
\sin \theta \cos \alpha & \cos \theta \cos \alpha & -\sin \alpha & -\sin \alpha \cdot d \\
\sin \theta \sin \alpha & \cos \theta \sin \alpha & \cos \alpha & \cos \alpha \cdot d \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Can be used, for example, as

\[T[\pi/2, 0, 0, \theta] //\text{MatrixForm}\]

which will print out transformation with those parameter values in a nice matrix form in the Mathematica notebook.

2. (Craig, 3.4)

Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 1.

Consulting the figure, the following link parameters are apparent:

<table>
<thead>
<tr>
<th>(i)</th>
<th>(a_{i-1})</th>
<th>(a_i)</th>
<th>(d_i)</th>
<th>(\theta_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\theta_1)</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{\pi}{2})</td>
<td>0</td>
<td>0</td>
<td>(\theta_2)</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>(L_3)</td>
<td>0</td>
<td>(\theta_3)</td>
</tr>
</tbody>
</table>

Determining \(0T_1^1\) and \(2^3T\) is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.
Figure 1. Frames for problem 2.
\[
\frac{1}{2} T = \begin{bmatrix}
\cos \theta_2 & -\sin \theta_2 & 0 & 0 \\
0 & 0 & -1 & 0 \\
\sin \theta_2 & \cos \theta_2 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\frac{2}{3} T = \begin{bmatrix}
\cos \theta_3 & -\sin \theta_3 & 0 & L_3 \\
\sin \theta_3 & \cos \theta_3 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

3. (Craig, 3.9)
We were given \( \frac{2}{3} T \), and want to find the location of the tip in frame 0.
Clearly,
\[
0 P_{\text{tip}} = \frac{1}{2} T \ 2 P_{\text{tip}},
\]
where
\[
2 P_{\text{tip}} = \begin{bmatrix}
l_2 \\
0 \\
0
\end{bmatrix},
\]
or
\[
0 P_{\text{tip}} = \begin{bmatrix}
l_2 \cos \theta_1 \cos \theta_2 + l_1 \cos \theta_1 \\
l_2 \sin \theta_1 \cos \theta_2 + l_1 \sin \theta_1 \\
l_2 \sin \theta_2
\end{bmatrix}.
\]

4. (Craig, 3.11)
Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 2.
Consulting the figure, the following link parameters are apparent:

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a_{i-1} )</th>
<th>( \alpha_{i-1} )</th>
<th>( d_i )</th>
<th>( \theta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \theta_4 )</td>
</tr>
<tr>
<td>5</td>
<td>( \phi )</td>
<td>0</td>
<td>0</td>
<td>( \theta_5 )</td>
</tr>
<tr>
<td>6</td>
<td>( -\phi )</td>
<td>0</td>
<td>0</td>
<td>( \theta_6 )</td>
</tr>
</tbody>
</table>

Determining \( B T, 4 T \) and \( 5 T \) is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.

\[ \text{<</home/bill/courses/me469/math/forward.m} \]
\[ \text{T[0,0,0,t4]. T[phi,0,0,t5]. T[-phi,0,0,t6] //Simplify} \]

produces the mess

3
Figure 2. Frames for problem 4.
\[
\begin{align*}
-(\sin[t4] \times & (\cos[\phi] \times \cos[t6] \times \sin[t5] + \\
& \cos[\phi]^2 \times \cos[t5] \times \sin[t6] + \sin[\phi]^2 \times \sin[t6])) + \\
& \cos[t4] \times (\cos[t5] \times \cos[t6] - \cos[\phi] \times \sin[t5] \times \sin[t6]), \\
& -(\cos[\phi]^2 \times \cos[t5] \times \sin[t4]) - \\
& \cos[t6] \times \sin[\phi]^2 \times \sin[t4] - \\
& \cos[t5] \times \sin[t5] - \cos[t4] \times \cos[t5] \times \sin[t6], \\
& -(\sin[\phi] \times (\cos[\phi] \times (-1 + \cos[t5]) \times \sin[t4] + \\
& \cos[t4] \times \sin[t5])), 0), \\
{\cos[\phi] \times \cos[t4 + t6] \times \sin[t5]} + \\
& \cos[t4] \times \sin[\phi]^2 \times \sin[t6] + \\
& \cos[t5] \times (\cos[t6] \times \sin[t4] + \cos[\phi]^2 \times \cos[t4] \times \sin[t6]), \\
& \cos[\phi]^2 \times \cos[t4] \times \cos[t5] \times \cos[t6] + \\
& \cos[t4] \times \cos[t5] \times \sin[t4 + t6], \\
& \sin[\phi] \times (\cos[\phi] \times \cos[t4] \times (-1 + \cos[t5]) - \\
& \sin[t4] \times \sin[t5]), 0), \\
{\sin[\phi] \times (\cos[t6] \times \sin[t5]} + \\
& \cos[\phi] \times (-1 + \cos[t5] \times \sin[t6]), \\
& \sin[\phi] \times (\cos[\phi] \times (-1 + \cos[t5]) \times \cos[t6] - \\
& \sin[t5] \times \sin[t6]), \cos[\phi]^2 \times \cos[t5] \times \sin[\phi] \times 2, 0), \\
{0, 0, 0, 1})
\end{align*}
\]

To check to make sure it makes sense, take all the joint angles, $\theta_4$, $\theta_5$ and $\theta_6$ to be zero:

```math
\text{\texttt{\$}}/\text{home/bill/courses/me469/math/forward.m}
T[0,0,0,0] \times T[\phi,0,0,0] \times T[-\phi,0,0,0] \text{ //Simplify //MatrixForm}
```

which gives the expected answer

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

If $\theta_4 = \pi$ and the other joints are zero, then

```math
\text{\texttt{\$}}/\text{home/bill/courses/me469/math/forward.m}
T[0,0,0,\Pi] \times T[\phi,0,0,0] \times T[-\phi,0,0,0] \text{ //Simplify //MatrixForm}
```

which also is expected because rotating joint 4 by $\pi$ will rotate the $x$ and $y$ components of a vector by $180^\circ$, which will make their components negative of what they start as.
Figure 3. Frames for problem 5.

5. (Craig, 3.17)
Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 3.

Consulting the figure, and naming the distance $a_2$, the following link parameters are apparent:

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\alpha_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>$\pi$</td>
<td>0</td>
<td>0</td>
<td>$\theta_2 + \frac{\pi}{2}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{\pi}{2}$</td>
<td>$a_2$</td>
<td>$d_3$</td>
<td>0</td>
</tr>
</tbody>
</table>

bf: Note the $\frac{\pi}{2}$ term added to $\theta_2$!

Determining $T_{12}T$ and $2T$ is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.

```
<</home/bill/courses/me469/math/forward.m
T[0,0,0,t1] . T[Pi/2,0,0,t2+Pi/2] . T[Pi/2,a2,d3,0] //Simplify
```
produces the mess

\[
\begin{pmatrix}
\cos t_1 \sin t_2, \sin t_1, \cos t_1 \cos t_2, \\
\cos t_1 \cos (d_3 \cos t_2 - a_2 \sin t_2), \\
-\sin t_1 \sin t_2, -\cos t_1, \cos t_2 \sin t_1, \\
\sin t_1 \cos (d_3 \cos t_2 - a_2 \sin t_2)
\end{pmatrix}, \\
0, 0, 0, 1
\]

or

\[
\begin{pmatrix}
-\cos t_1 \sin t_2 & \sin t_1 & \cos t_1 \cos t_2 & \cos t_1 \left( d_3 \cos t_2 - a_2 \sin t_2 \right) \\
-\sin t_1 \sin t_2 & -\cos t_1 & \cos t_2 \sin t_1 & \sin t_1 \left( d_3 \cos t_2 - a_2 \sin t_2 \right) \\
\cos t_2 & 0 & \sin t_2 & a_2 \cos t_2 + d_3 \sin t_2 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

To check to make sure it makes sense, take all the joint angles, \( \theta_1, \theta_2 \) and \( d_3 \) to be zero:

```math
<< /home/bill/courses/me469/math/forward.m
t[0,0,0,0] . t[pi/2,0,0,0] . t[pi/2,a2,0,0] //simplify
```

which gives the expected answer

\[
\begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
1 & 0 & 0 & a_2 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

6. (Craig, 3.18)

Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 4.

Consulting the figure, and naming the distance \( a_2 \), the following link parameters are apparent:

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \alpha_{i-1} )</th>
<th>( a_{i-1} )</th>
<th>( d_i )</th>
<th>( \theta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \theta_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{\pi}{2} )</td>
<td>0</td>
<td>0</td>
<td>( \theta_2 + \frac{\pi}{2} )</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>( a_2 )</td>
<td>0</td>
<td>( \theta_3 )</td>
</tr>
</tbody>
</table>

bf: Note the \( \frac{\pi}{2} \) term added to \( \theta_2 \)!

Determining \( 0T_{12}T \) and \( 3T \) is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.

```math
<< /home/bill/courses/me469/math/forward.m
t[0,0,0,t1] . t[pi/2,0,0,t2+pi/2] . t[pi/2,a2,d3,0] //simplify
```

7
Figure 4. Frames for problem 6.
gives

\[
\begin{bmatrix}
-\cos \theta_1 \sin(\theta_2 + \theta_3) & -\cos \theta_1 \cos(\theta_2 + \theta_3) & \sin \theta_1 & -a_2 \cos \theta_1 \sin \theta_2 \\
-\sin \theta_1 \sin(\theta_2 + \theta_3) & -\cos(\theta_2 + \theta_3) \sin \theta_1 & \cos \theta_1 & -a_2 \sin \theta_1 \sin \theta_2 \\
\cos(\theta_2 + \theta_3) & -\sin(\theta_2 + \theta_3) & 0 & a_2 \cos \theta_2 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

To check to make sure it makes sense, take all the joint angles, \( \theta_1, \theta_2 \) and \( \theta_3 \) to be zero:

```plaintext
<<home/bill/courses/me469/math/forward.m
T[0,0,0,0] . T[Pi/2,0,0,Pi/2] . T[0,a2,0,0] //Simplify
```

which gives the expected answer

\[
\begin{bmatrix}
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
1 & 0 & 0 & \ a_2 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

7. (Craig, 3.20)

Following the usual rules for affixing frames to manipulators results in the frames illustrated in Figure 5.

Consulting the figure, and naming the distances \( a_1 \) and \( a_2 \), the following link parameters are apparent:

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a_{i-1} )</th>
<th>( a_i )</th>
<th>( d_i )</th>
<th>( \theta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( d_1 )</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>( a_1 )</td>
<td>0</td>
<td>( \theta_2 )</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>( a_2 )</td>
<td>0</td>
<td>( \theta_3 )</td>
</tr>
</tbody>
</table>

Determining \( 0_{i-1} \) \( T_i \) and \( 3_3 \) \( T \) is now a simple matter of substituting these values into Equation 3.6, or, even simpler, using the Mathematica function from Problem 1.

```plaintext
<<home/bill/courses/me469/math/forward.m
T[0,0,d1,0] . T[0,a1,0,t2] . T[0,a2,0,t3] //Simplify
```

gives

\[
\begin{bmatrix}
\cos(\theta_2 + \theta_3) & -\sin(\theta_2 + \theta_3) & 0 & a_1 + \cos(\theta_2) a_2 \\
\sin(\theta_2 + \theta_3) & \cos(\theta_2 + \theta_3) & 0 & \sin(\theta_2) a_2 \\
0 & 0 & 1 & \ d_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Figure 5. Frames for problem 7.