University of Notre Dame
Aerospace and Mechanical Engineering

ME 469: Introduction to Robotics
Homework 3 Solutions

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1. (Craig, 3.16)

The link frame assignments are illustrated in Figure 1 (any axis not shown is determined by the right hand rule).

Consulting the figure, and naming the distances a_1 and a_2, the following link parameters are apparent:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\frac{\pi}{2}$	a_1	$d_2 + a_2$	0
3	$\frac{\pi}{2}$	0	0	θ_3

Note that I took the the reference value for θ_3 to be the position of the wrist when it is "straight out," and the reference value for d_2 to be zero when the frame 3 is located a distance of a_2 away from frame 2.

Plugging into Equation 3.6, or the Mathematica function gives the forward kinematics

$$
{}^0_3T = \begin{bmatrix} \cos(\theta_3 - \theta_1) & -\sin(\theta_3 - \theta_1) & 0 & a_1 \cos(\theta_1) + (d_2 + a_2)\sin(\theta_1) \\ -\sin(\theta_3 - \theta_1) & -\cos(\theta_3 - \theta_1) & 0 & -((d_2 + a_2)\cos(\theta_1)) + a_1 \sin(\theta_1) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

2. (Craig, 3.19)

For this problem, take the zero position of the links as follows:

- for link 2, let $d_2 = 0$ when the link is all the way "down" and
- for link 3, let $d_3 = 0$ when the wrist is flush against the joint stop, *i.e.*, all the way to the left.

The link frame assignments are illustrated in Figure 2.

Consulting the figure and naming the distance a_2, the following link parameters are apparent:

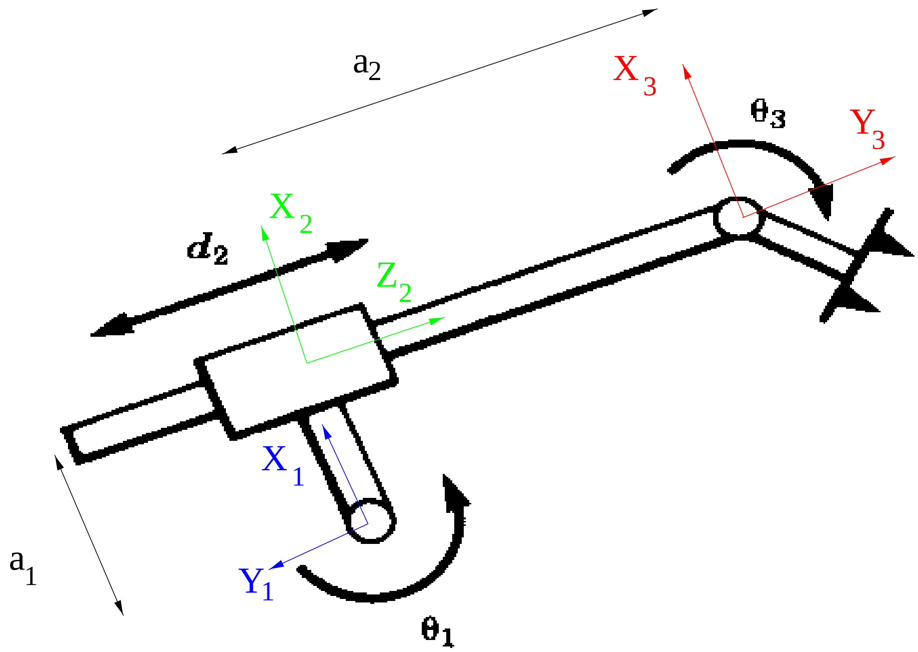

Figure 1. Link frame assignments for problem 1.

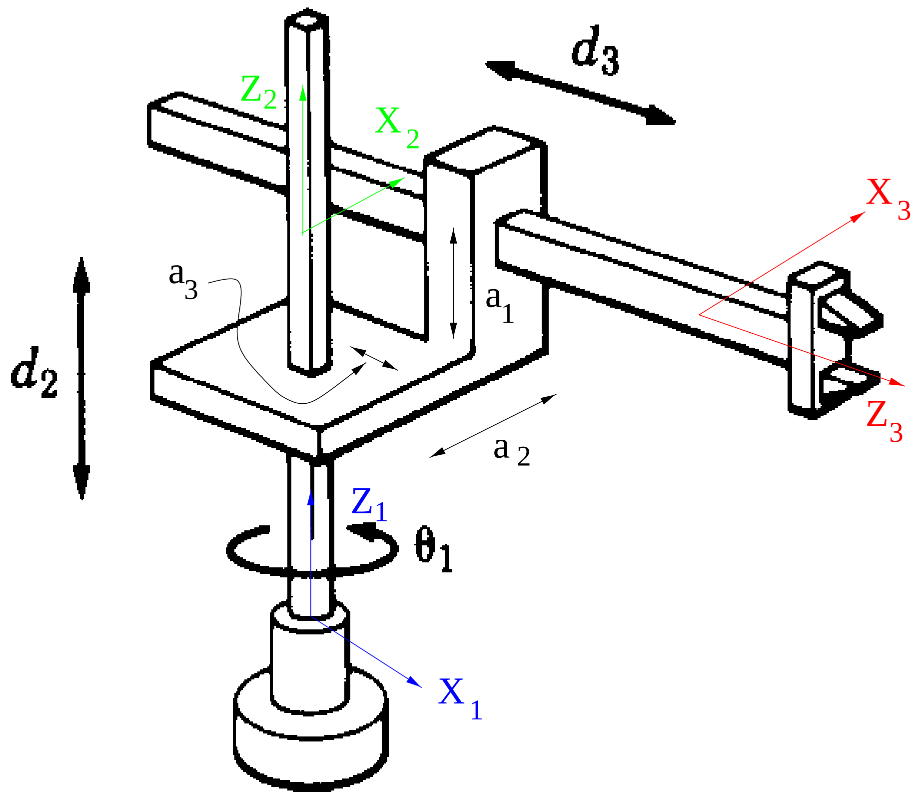

Figure 2. Link frame assignments for problem 2.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	0	$a_1 + d_2$	$\frac{\pi}{2}$
3	$\frac{\pi}{2}$	a_2	$d_3 + a_3$	0

Plugging into Equation 3.6, or the mathematica function gives the forward kinematics

$$
{}^0_3T = \begin{bmatrix} -\sin\theta_1 & 0 & \cos\theta_1 & (d_3 + a_3)\cos\theta_1 - a_2 \sin\theta_1 \\ \cos\theta_1 & 0 & \sin\theta_1 & a_2\cos\theta_1 + (d_3 + a_3)\sin\theta_1 \\ 0 & 1 & 0 & a_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

3. (Craig, 3.21)

For this problem, take the zero position of the links as follows:

- for link 1, let $d_1 = 0$ in the middle (where pictured);
- for link 2, let $d_2 = 0$ when the link is in the middle (to the left of where pictured); and,
- for link 3, let $d_3 = 0$ when the link is all the way "down."

Note that I picked joint axis 1 in the middle of the mechanism.

The link frame assignments are illustrated in Figure 3.

Consulting the figure, the following link parameters are apparent:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	0
2	$-\frac{\pi}{2}$	0	d_2	$-\frac{\pi}{2}$
3	$-\frac{\pi}{2}$	0	d_3	0

Plugging into Equation 3.6, or the Mathematica function gives the forward kinematics

$$
{}^0_3T = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

4. (Craig, 5.12)

This problem could be done intuitively:

- Workspace boundary singularity: at $\theta_2 = \theta_3 = 0$.
- Interior singularity: at $\theta_2 = \pi$.

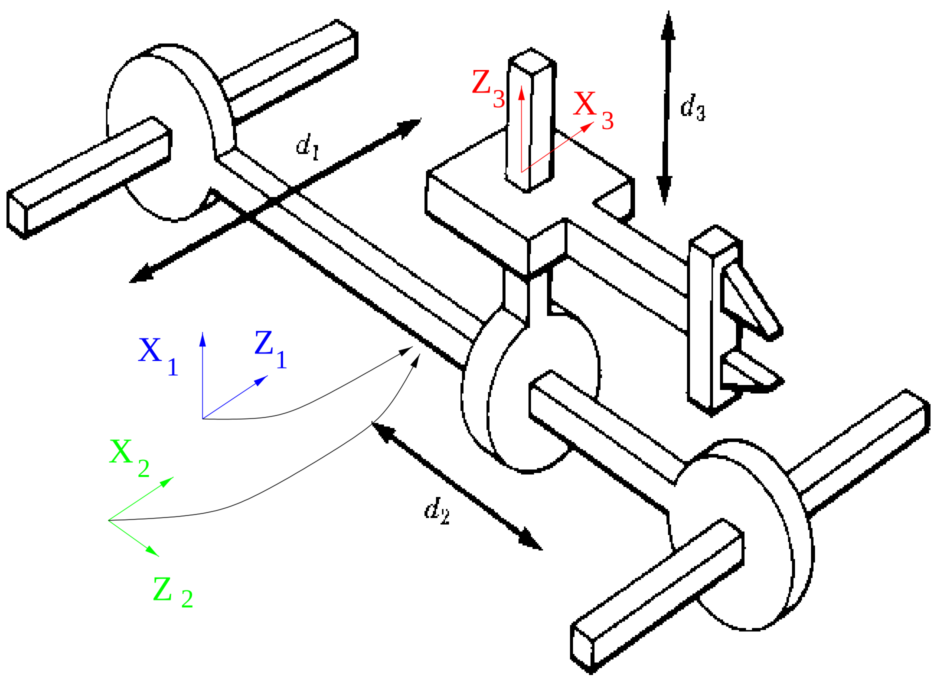

Figure 3. Link frame assignments for problem 3.

5. (Craig, 5.18)
This is just asking for the linear part, so all we have to do is differentiate the top three terms of the fourth column:

$$
{}^0P_{3ORG} = \left[\begin{array}{c} l_1c_1 + l_2c_1c_2 \\ l_1s_1 + l_2s_1c_2 \\ l_2s_2 \end{array} \right],
$$

therefore

$$
{}^0\dot{P}_{3ORG} = \left[\begin{array}{c} -\dot{\theta}_1(l_1s_1 + l_2s_1c_2) - \dot{\theta}_2l_2c_1s_2 \\ \dot{\theta}_1(l_1c_1 + l_2c_1c_2) - \dot{\theta}_2l_2s_1s_2 \\ \dot{\theta}_2l_2c_2 \end{array} \right],
$$

or, in matrix form,

$$
\boxed{ {}^0J(\Theta) = \left[\begin{array}{ccc} -l1s_1 - l_2s_1c_2 & -l_2c_1s_2 & 0 \\ l_1c_1 + l_2c_1c_2 & -l_2s_1s_2 & 0 \\ 0 & l_2c_2 & 0 \end{array} \right]. }
$$

6. (Craig, 5.19)
Using the general definition of a Jacobian directly gives:

$$
J(\Theta) = \left[\begin{array}{cc} -a_1s_1 - d_2c_1 & -s_1 \\ a_1c_1 - d_2s_1 & c_1 \end{array} \right].
$$

Now,

$$
\det\left(J(\Theta)\right) = -d_2.
$$

Therefore, the manipulator is at a singular configuration when

$$
d_2 = 0.
$$