University of Notre Dame
Aerospace and Mechanical Engineering

ME 469: Introduction to Robotics
Midterm Exam

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NAME: ______________________________

- You have 180 minutes to complete this exam, but it is designed to take 60 – 90 minutes.
- This is an open book exam. You may consult the course text, your class notes, your own homework sets and any documents provided on the course homepage such as homework solutions, tables, etc.
- There are 3 questions. Problem 1 is worth 25 points, Problem 2 is worth 60 points and Problem 3 is worth 15 points.
- Your grade on this exam will constitute 15% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

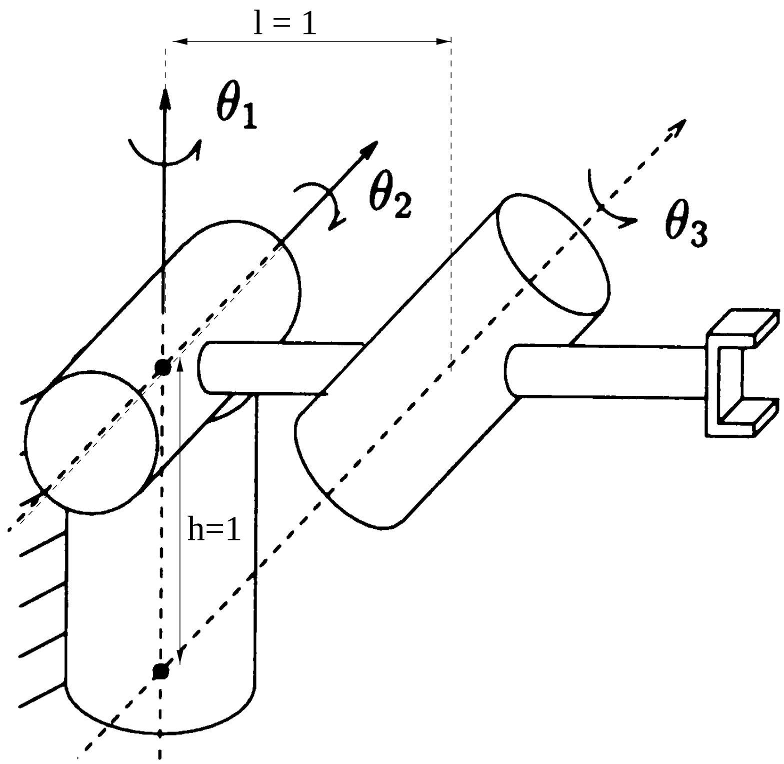


Figure 1. Manipulator for Problem 1.

1. (25 points)

   Consider the manipulator illustrated in Figure 1.

   Directly on the Figure, draw the location of each link frame, attached to the manipulator in accordance with the method outlined in the course text and in class. (15 points)

   For each link, determine the link parameters and complete the following chart. (10 points)

i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				
3				

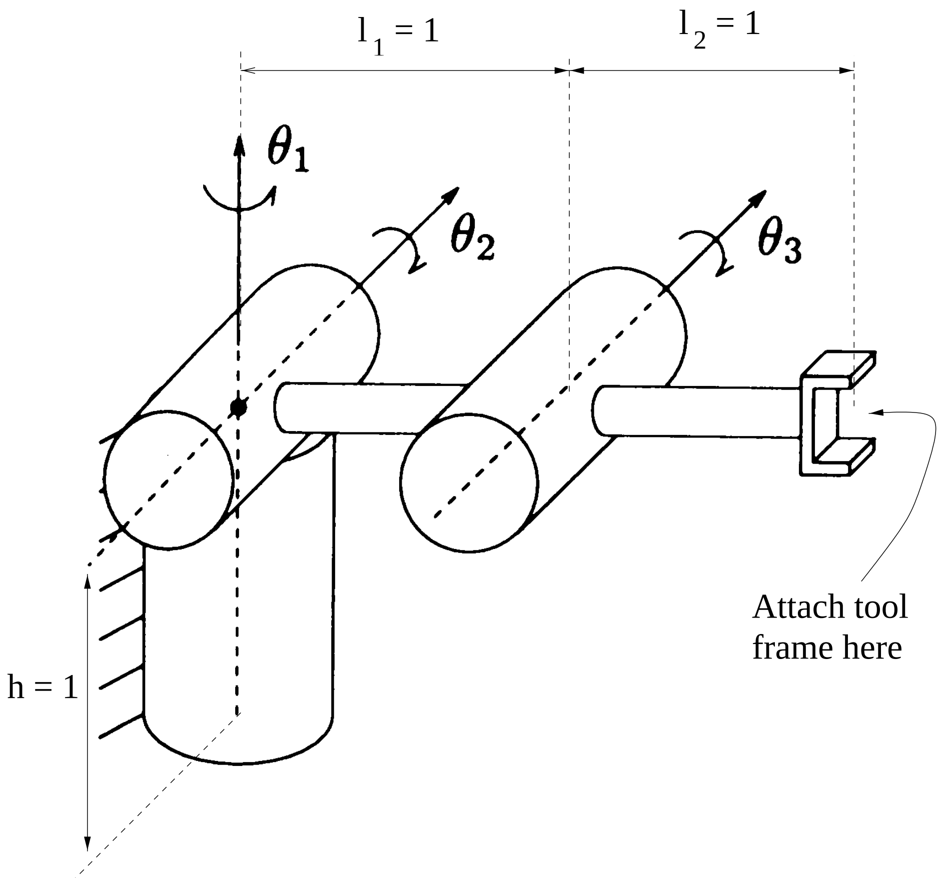


Figure 2. Manipulator for Problem 2.

2. (60 points)

   Consider the manipulator illustrated in Figure 2.

   Directly on the Figure, draw the location of each link frame, attached to the manipulator in accordance with the method outlined in the course text and in class. (10 points)

   For each link, determine the link parameters and complete the following chart. (10 points)

i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				
3				

Determine ${}^{0}_{1}T$, ${}^{1}_{2}T$ and ${}^{2}_{3}T$. (10 points)

Write the answers (4×4 matrices) in the space provided:

${}^{0}_{1}T =$

${}^{1}_{2}T =$

${}^{2}_{3}T =$

Now, add a tool frame, T attached to the end effector, and determine ${}^3_T T$. Assuming we are interested only in the (x, y, z) position variables of the end effector (the origin of frame T), determine the Jacobian which related the input joint velocities to the linear velocities, *i.e.*, determine J such that

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix},$$

where (p_x, p_y, p_z) is the location of the origin of frame 3 written in frame 1, *i.e.*, ${}^0P_{TORG}$.

(20 points)

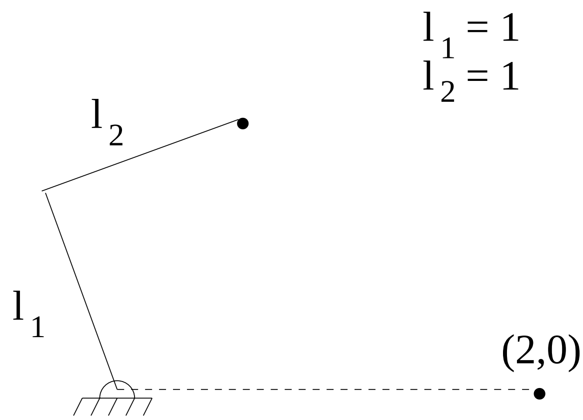
Determine at least one singular configuration for the manipulator illustrated in Figure 2.

(10 points)

3. (15 points)

Answer each of the following. Two of the five points are for getting (guessing) the correct yes/no or true/false answer. Three points are for providing a correct justification.

(a) Consider the two link robot illustrated below. Is it possible for the robot to draw a straight line from the origin to the point $(2, 0)$ even though the Jacobian is singular at the end points of the motion? Why or why not? (5 points)



(b) Appendix B of the book provides the matrix representations for the 12 Euler angle sets. True or False: If

$$R = R_{X'Y'Z'}(\alpha, \beta, \gamma),$$

then

$$R^{-1} = R_{X'Y'Z'}(-\alpha, -\beta, -\gamma).$$

(5 points)

(c) In class, we considered the matrix exponential as a representation of rigid body rotation,

$$R = e^{\hat{\omega}},$$

where $\hat{\omega}$ is skew–symmetric, *i.e.*, $\hat{\omega}^T = -\hat{\omega}$. True or false: matrix exponentials obey the usual rules for multiplying exponentials:

$$e^{\hat{\omega}_1} e^{\hat{\omega}_2} = e^{\hat{\omega}_1 + \hat{\omega}_2}.$$

Justify your answer. (5 points)