ME469 HW #6 Solution

```
In[2]:= << LinearAlgebra`MatrixManipulation`;
        T[α_, a_, d_, θ_] := {{Cos[θ], -Sin[θ], 0, a}, {Sin[θ] * Cos[α], Cos[θ] * Cos[α], -Sin[α], -Sin[α] * d},
          {Sin[θ] * Sin[α], Cos[θ] * Sin[α], Cos[α], Cos[α] * d}, {0, 0, 0, 1}};

        # Prob 1

In[3]:= T04 = T[0, 0, 0, θ1] . T[Pi / 2, 0, 0, θ2] . T[0, a2, 0, θ3] . T[0, a3, 0, 0];
        MatrixForm[T04 // Simplify]
```

$$\begin{pmatrix} \text{Cos}[\theta_1]\,\text{Cos}[\theta_2+\theta_3] & -\text{Cos}[\theta_1]\,\text{Sin}[\theta_2+\theta_3] & \text{Sin}[\theta_1] & \text{Cos}[\theta_1]\,(\text{a2}\,\text{Cos}[\theta_2]+\text{a3}\,\text{Cos}[\theta_2+\theta_3]) \\ \text{Cos}[\theta_2+\theta_3]\,\text{Sin}[\theta_1] & -\text{Sin}[\theta_1]\,\text{Sin}[\theta_2+\theta_3] & -\text{Cos}[\theta_1] & (\text{a2}\,\text{Cos}[\theta_2]+\text{a3}\,\text{Cos}[\theta_2+\theta_3])\,\text{Sin}[\theta_1] \\ \text{Sin}[\theta_2+\theta_3] & \text{Cos}[\theta_2+\theta_3] & 0 & \text{a2}\,\text{Sin}[\theta_2]+\text{a3}\,\text{Sin}[\theta_2+\theta_3] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[4]:= j11 = D[T04[[1]][[4]], θ1]; j12 = D[T04[[1]][[4]], θ2]; j13 = D[T04[[1]][[4]], θ3];
        j21 = D[T04[[2]][[4]], θ1]; j22 = D[T04[[2]][[4]], θ2]; j23 = D[T04[[2]][[4]], θ3];
        j31 = D[T04[[3]][[4]], θ1]; j32 = D[T04[[3]][[4]], θ2]; j33 = D[T04[[3]][[4]], θ3];
        Jacobian1 = Simplify[{{j11, j12, j13}, {j21, j22, j23}, {j31, j32, j33}}];
        MatrixForm[Jacobian1]
```

Out[4]//MatrixForm=

$$\begin{pmatrix} -(\text{a2}\,\text{Cos}[\theta_2]+\text{a3}\,\text{Cos}[\theta_2+\theta_3])\,\text{Sin}[\theta_1] & -\text{Cos}[\theta_1]\,(\text{a2}\,\text{Sin}[\theta_2]+\text{a3}\,\text{Sin}[\theta_2+\theta_3]) & -\text{a3}\,\text{Cos}[\theta_1]\,\text{Sin}[\theta_2+\theta_3] \\ \text{Cos}[\theta_1]\,(\text{a2}\,\text{Cos}[\theta_2]+\text{a3}\,\text{Cos}[\theta_2+\theta_3]) & -\text{Sin}[\theta_1]\,(\text{a2}\,\text{Sin}[\theta_2]+\text{a3}\,\text{Sin}[\theta_2+\theta_3]) & -\text{a3}\,\text{Sin}[\theta_1]\,\text{Sin}[\theta_2+\theta_3] \\ 0 & \text{a2}\,\text{Cos}[\theta_2]+\text{a3}\,\text{Cos}[\theta_2+\theta_3] & \text{a3}\,\text{Cos}[\theta_2+\theta_3] \end{pmatrix}$$

```
In[5]:= Jacobian1 = N[Jacobian1 /. {a2 -> 1., a3 -> 1., θ1 -> Pi / 6., θ2 -> Pi / 6., θ3 -> Pi / 6.}];
        Eigensystem[Jacobian1 . Transpose[Jacobian1]]

Out[5]= {{4.67862, 1.86603, 0.0534346},
          {{-0.644623, -0.372173, 0.667793}, {0.5, -0.866025, -2.70589 × 10^-16}, {0.578326, 0.333897, 0.744347}}}
```

The smallest eigenvalue is the 3 rd one , thus the 3 rd eigenvalue is the direction of maximum mechanical advantage;

The biggest eigenvalue is the 1 st one, thus the 1 st eigenvalue is the direction of maximum velocity amplitude;

```
        # Prob 2
```

```
In[6]:= T04 = T[0, 0, d1, 0] . T[0, a1, 0, θ2] . T[0, a2, 0, θ3] . T[0, a3, 0, 0] // Simplify;
        MatrixForm[T04 // Simplify]
```

Out[6]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta_2+\theta_3] & -\text{Sin}[\theta_2+\theta_3] & 0 & \text{a2 Cos}[\theta_2]+\text{a3 Cos}[\theta_2+\theta_3]+\text{a}_1 \\ \text{Sin}[\theta_2+\theta_3] & \text{Cos}[\theta_2+\theta_3] & 0 & \text{a2 Sin}[\theta_2]+\text{a3 Sin}[\theta_2+\theta_3] \\ 0 & 0 & 1 & \text{d}_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[7]:= j11 = D[T04[[1]][[4]], θ1]; j12 = D[T04[[1]][[4]], θ2]; j13 = D[T04[[1]][[4]], θ3];
        j21 = D[T04[[2]][[4]], θ1]; j22 = D[T04[[2]][[4]], θ2]; j23 = D[T04[[2]][[4]], θ3];
        j31 = D[T04[[3]][[4]], θ1]; j32 = D[T04[[3]][[4]], θ2]; j33 = D[T04[[3]][[4]], θ3];
        Jacobian2 = Simplify[{{j11, j12, j13}, {j21, j22, j23}, {j31, j32, j33}}];
        MatrixForm[Jacobian2]
```

Out[7]//MatrixForm=

$$\begin{pmatrix} 0 & -\text{a2 Sin}[\theta_2]-\text{a3 Sin}[\theta_2+\theta_3] & -\text{a3 Sin}[\theta_2+\theta_3] \\ 0 & \text{a2 Cos}[\theta_2]+\text{a3 Cos}[\theta_2+\theta_3] & \text{a3 Cos}[\theta_2+\theta_3] \\ 0 & 0 & 0 \end{pmatrix}$$

```
In[8]:= Jacobian2 = N[Jacobian2 /. {a2 -> 1., a3 -> 1., θ2 -> Pi / 6., θ3 -> Pi / 6.}];
        Eigensystem[Jacobian2 . Transpose[Jacobian2]]

Out[8]= {{4.67862, 0.0534346, 0.}, {{0.744347, -0.667793, 0.}, {0.667793, 0.744347, 0.}, {0., 0., 1.}}}
```

The smallest eigenvalue is the 3 rd one , thus the 3 rd eigenvalue is the direction of worst velocity amplitude;

The biggest eigenvalue is the 1 st one, thus the 1 st eigenvalue is the direction of worst mechanical advantage;

```
        # Prob 3

In[9]:= T04 = T[0, 0, d1, 0] . T[-Pi / 2, 0, d2, -Pi / 2] . T[-Pi / 2, 0, d3, 0] . T[0, d4, 0, 0] // Simplify;
        MatrixForm[T04 // Simplify]
```

Out[9]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & \text{d}_3 \\ 0 & -1 & 0 & \text{d}_2 \\ 1 & 0 & 0 & \text{d}_1+\text{d}_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[10]:= j11 = D[T04[[1]][[4]], d1]; j12 = D[T04[[1]][[4]], d2]; j13 = D[T04[[1]][[4]], d3];
         j21 = D[T04[[2]][[4]], d1]; j22 = D[T04[[2]][[4]], d2]; j23 = D[T04[[2]][[4]], d3];
         j31 = D[T04[[3]][[4]], d1]; j32 = D[T04[[3]][[4]], d2]; j33 = D[T04[[3]][[4]], d3];
         Jacobian3 = Simplify[{{j11, j12, j13}, {j21, j22, j23}, {j31, j32, j33}}];
         MatrixForm[Jacobian3]
```

Out[10]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

```
In[11]:= Eigensystem[Jacobian3 . Transpose[Jacobian3]]

Out[11]= {{1, 1, 1}, {{0, 0, 1}, {0, 1, 0}, {1, 0, 0}}}
```

The eigenvalues are the same . This indicates that there is no preferred direction of Maximum force fidelity and maximum velocity fidelity .