University of Notre Dame
Aerospace and Mechanical Engineering

ME 469: Introduction to Robotics
Midterm Exam

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NAME: ______________________________

- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, your class notes, your own homework sets and any documents provided on the course homepage such as homework solutions, tables, etc.
- There are 3 questions. Problem 1 is worth 60 points, Problem 2 is worth 25 points and Problem 3 is worth 15 points.
- Your grade on this exam will constitute 15% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- Do not start or turn the page until instructed to do so.

1. Consider the robotic manipulator illustrated in Figure 1.

   (a) Directly on the figure, draw the location and orientation of each link frame, (frames {0} through {6}) attached to the manipulator in accordance with the Denavit–Hartenberg method outlined in class and in the course text. (20 points)

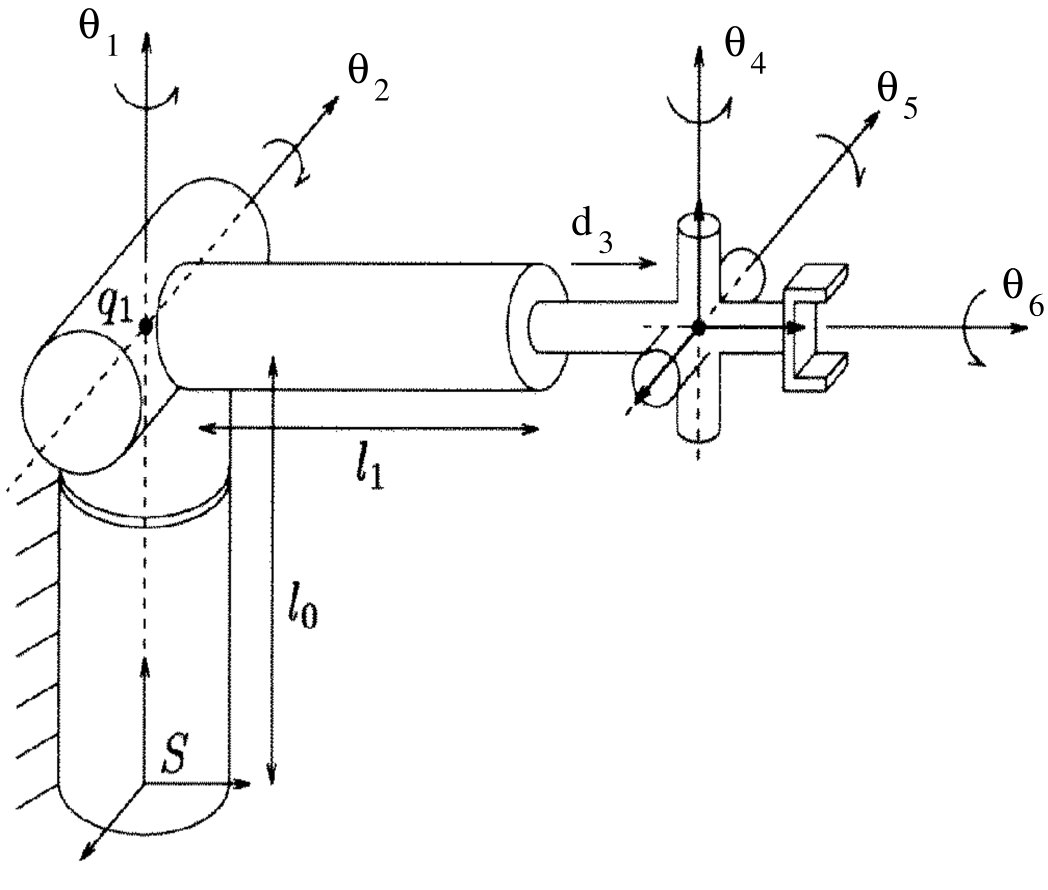

Figure 1. Robotic Manipulator for Problem 1.

(b) For each link, determine the link parameters,and complete the following chart. (20 points)

i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				
3				
4				
5				
6				

(c) What is ${}^2_3T$? (10 points)

(d) What is ${}^5_6T$? (10 points)

2. In the book, equations 3.16 and 3.17 on pages 97 and 98 give expressions for the forward kinematics of the Yasukawa Motoman L–3 robot.

   (a) Using these equations, determine the Jacobian that relates joint velocities to the *linear* velocity of the origin of frame 5, *i.e.*, find J such that

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \end{bmatrix}.$$

(15 points)

(b) Concisely explain in terms that an Arts & Letters major could understand what a singular configuration of a robot is and what the effects are on robot control when a robot is near or at a singular configuration. Specifically discuss end effector velocities, forces, joint torques and joint velocities. (10 points)

3. (15 points)

Answer each of the following. Two of the five points are for getting (guessing) the correct yes/no or true/false answer. Three points are for providing a correct justification.

(a) Appendix B of the book provides the matrix representations for the 12 Euler angle sets. True or False: If

$$ {}^{A}_{B}R = R_{X'Y'Z'}(\alpha_1, \beta_1, \gamma_1), $$

and

$$ {}^{B}_{C}R = R_{X'Y'Z'}(\alpha_2, \beta_2, \gamma_2), $$

then

$$ {}^{A}_{C}R = R_{X'Y'Z'}(\alpha_1 + \alpha_2, \beta_1 + \beta_2, \gamma_1 + \gamma_2). $$

Justify your answer. (5 points)

(b) In class, we considered the matrix exponential as a representation of rigid body rotation,

$$R = e^{\hat{\omega}},$$

where $\hat{\omega}$ is skew–symmetric, *i.e.*, $\hat{\omega}^T = -\hat{\omega}$.
For scalars we know that

$$e^a e^b = e^{a+b},$$

e.g., $e^2 e^3 = e^5$.
True or false: matrix exponentials of skew–symmetric matricies obey the same rules for multiplying exponentials:

$$e^{\hat{\omega}_1} e^{\hat{\omega}_2} = e^{\hat{\omega}_1 + \hat{\omega}_2}.$$

Justify your answer. (5 points)

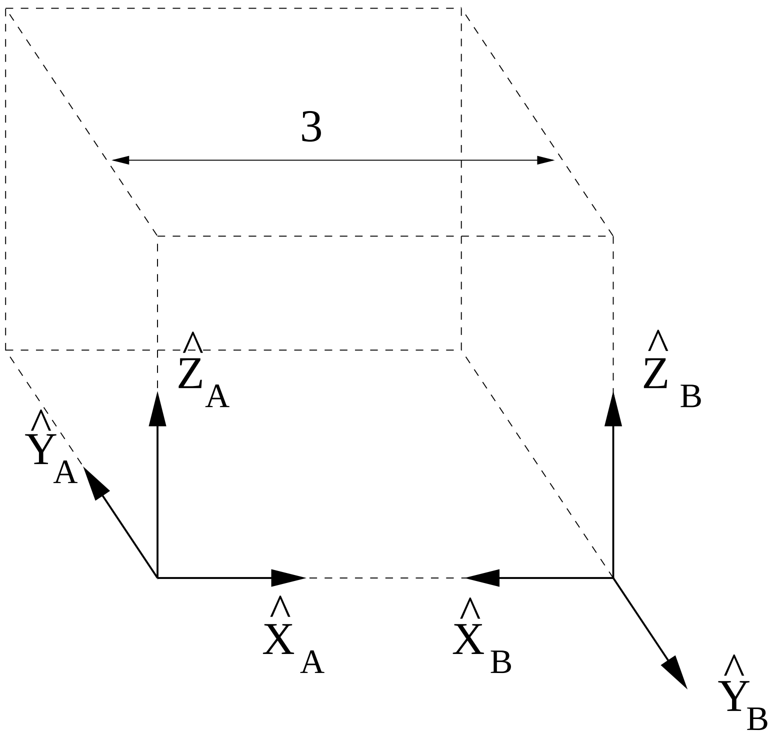

Figure 2. Frames for Problem 3c.

(c) Figure 2 illustrates two frames attached to two corners of a rectangular prism. What is ${}^A_BT$? (5 points)