

Properties of Laplace Transforms

Name	Time Function	Laplace Transform
Transform pair	$f(t)$	$F(s)$
Superposition	$\alpha f_1(t) + \beta f_2(t)$	$\alpha F_1(s) + \beta F_2(s)$
Time delay ($\tau \geq 0$)	$f(t - \tau)$	$F(s)e^{-s\tau}$
Time scaling	$f(at)$	$\frac{1}{ a }F\left(\frac{s}{a}\right)$
Frequency shift	$e^{-at}f(t)$	$F(s + a)$
Differentiation	$\frac{d^m}{dt^m}f(t)$	$s^m F(s) - s^{m-1}f(0) - \dots - \frac{d^{m-1}}{dt^{m-1}}f(0)$
Integration	$\int_s f(\xi)d\xi$	$\frac{1}{s}F(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Initial Value Theorem	$f(0^+)$	$\lim_{s \rightarrow \infty} sF(s)$
Final Value Theorem	$\lim_{t \rightarrow \infty} f(t)$	$\lim_{s \rightarrow 0} sF(s)$
Time product	$f_1(t)f_2(t)$	$\frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F_1(\xi)F_2(s - \xi)d\xi$
Multiplication by time	$tf(t)$	$-\frac{d}{ds}F(s)$