University of Notre Dame
Aerospace and Mechanical Engineering

AME 301: Differential Equations, Vibrations and Controls I
Exam 1

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NAME: ______________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course texts, any other text book, your class notes, homework solutions and your own homework sets. You may **not** use a calculator.
- There are four problems, each is worth 25 points.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

"It is ever so with the things that Men begin: there is a frost in Spring, or a blight in Summer, and they fail of their promise," said Gimli.

Yet seldom do they fail of their seed," said Legolas. "And that will lie in the dust and rot to spring up again in times and places unlooked-for. The deeds of Men will outlast us, Gimli."

"And yet come to naught in the end but might-have-beens, I guess," said the Dwarf.

"To that, the Elves do not know the answer," said Legolas.

— J.R.R. Tolkien, *The Lord of the Rings*

1. Find $x(t)$ such that

$$\begin{aligned} \dot{x} - 2x &= 5 \\ x(0) &= -\frac{3}{2}. \end{aligned}$$

(25 points)

2. Find $x(t)$ such that

$$\begin{aligned} \ddot{x} - 4\dot{x} + 4x &= 2e^{2t} \\ x(0) &= 0 \\ \dot{x}(0) &= 0. \end{aligned}$$

(25 points)

3. Consider

$$\begin{aligned} \ddot{x} - 4\dot{x} + 4x &= 2e^{2t} \\ x(0) &= 0 \\ \dot{x}(0) &= 0. \end{aligned} \tag{1}$$

(a) Write this second order equation as a system of two first order equations. (5 points)

(b) The following code is used to determine an approximate numerical solution to Equation 1 using Euler's method. Fill in the blanks. (10 points)

```
#include<stdio.h>
#include<math.h>

main() {
  double t,dt,x1,x2;
  double t_start=0.0,t_finish=25.0;
  FILE *fp;

  dt = 0.01;
  x1 = 0.0;
  x2 = 0.0;

  fp = fopen("data.d","w");

  for(t=t_start;t<=t_finish;t+=dt) {
    fprintf(fp,"%.3f\t%.3f\t%.3f\n",t,x1,x2);
    x1 += ____________________________________;
    x2 += ____________________________________;
  }

  fclose(fp);
}
```

(c) The following code is used to determine an approximate numerical solution to Equation 1 using the 4th order Runge-Kutta method. Fill in the blanks. (10 points)

```
#include<stdio.h>
#include<math.h>

main() {
  double t,dt,x1,x2;
  double k1,k2,k3,k4,l1,l2,l3,l4;
  double t_start=0.0,t_finish=25.0;
  FILE *fp;

  dt = 0.1;
  x1 = 0.0;
  x2 = 0.0;

  fp = fopen("data.d","w");

  for(t=t_start;t<=t_finish;t+=dt) {
    fprintf(fp,"%.3f\t%.3f\t%.3f\n",t,x1,x2);

    k1 = ____________________________________;
    l1 = ____________________________________;

    k2 = ____________________________________;
    l2 = ____________________________________;

    k3 = ____________________________________;
    l3 = ____________________________________;

    k4 = ____________________________________;
    l4 = ____________________________________;

    x1 += ____________________________________;
    x2 += ____________________________________;
  }

  fclose(fp);
}
```

4. Figure 1 is a plot of the solution to

$$\begin{aligned} m\ddot{x} + b\dot{x} + kx &= F_0 \cos \omega t \\ x(0) &= 0 \\ \dot{x}(0) &= 0 \end{aligned} \tag{2}$$

where $m = 1$, $b = 0.5$, $k = 1$, $\omega = 1$ and $F_0 = 1$.

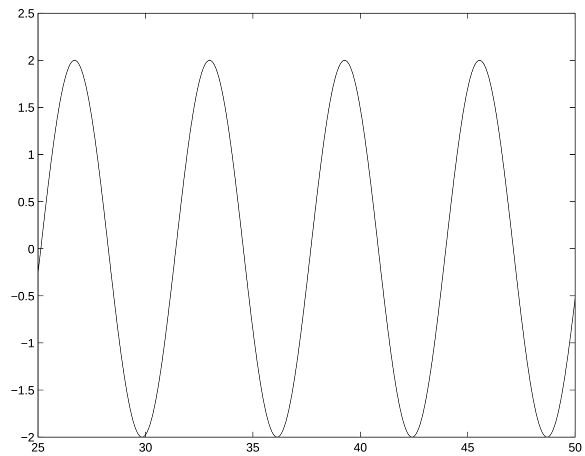

Figure 1. Response of system in equation 2.

Match each of the following four cases with the plots illustrated in Figures 2 through 5 and explain your reasoning. (6 points each — 1 free point!)

- In each case only one of the parameters is different from the system given above and is indicated in **bold** type.
- It is acceptable that your reasoning be based upon a comparison between the given figures and Figure 1. It is not necessary to use any equations in your explanation, but it may be helpful.
- The time axis starts at $t = 25$ in each figure; therefore, it is acceptable to only consider the particular solution. By the time $t = 25$ the effect of the homogeneous solution is negligible.
- Note that scale of the axis of the ordinate varies dramatically among the various figures.

(a) Figure ______________ is where $m = 1$, $b = 0.5$, $k = 1$, $\omega = \mathbf{2}$ and $F_0 = 1$ because:

(b) Figure ______________ is where $m = 1$, $b = 0.5$, $k = 1$, $\omega = 1$ and $\mathbf{F_0} = \mathbf{2}$. because:

(c) Figure ______________ is where $m = 1$, $b = 0.5$, $\mathbf{k} = \mathbf{2}$, $\omega = 1$ and $F_0 = 1$. because:

(d) Figure ______________ is where $m = 1$, $\mathbf{b} = \mathbf{0.0}$, $k = 2$, $\omega = 1$ and $F_0 = 1$. because:

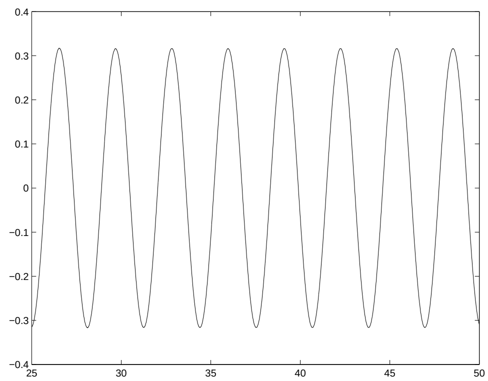

Figure 2.

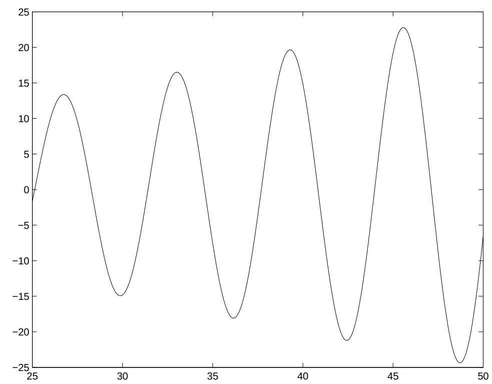

Figure 3.

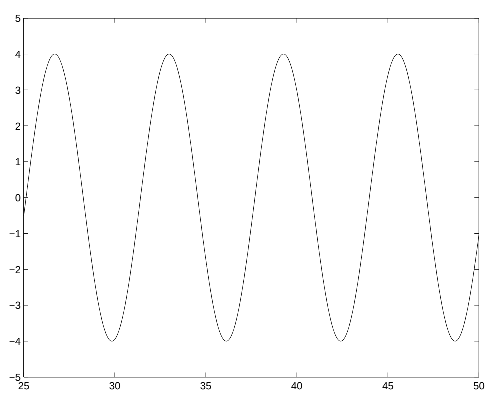

Figure 4.

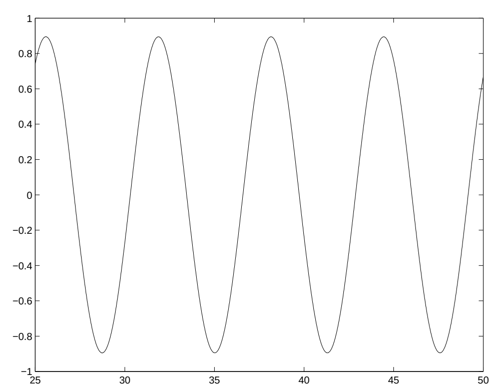

Figure 5.