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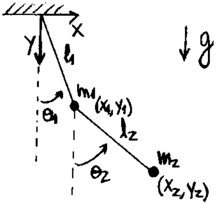

Lagrangian L of the system is $L = T$ ← Kinetic Energy

$\Rightarrow L = T_1 + T_2$

$$\Rightarrow L = \frac{m_1}{2} l_1^2 \dot{\theta}_1^2 + \frac{m_2}{2} l_2^2 \dot{\theta}_2^2 + \frac{m_2}{2} l_1^2 \dot{\theta}_1^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

Euler - Lagrange Equations

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i} \quad (1) \qquad q_i : \theta_1 \ \& \ \theta_2$$

$Q_i \neq 0$ since we have the gravity force.
$(g \neq 0)$

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$$Q_i = \sum_j F_j \cdot \frac{\partial r_j}{\partial \theta_i}$$

m_1: $F_1 = m_1 g \hat{j}$, $F_2 = m_2 g \hat{j}$

$$\frac{\partial r_1}{\partial \theta_1} = l_1 \cos\theta_1 \hat{\imath} - l_1 \sin\theta_1 \hat{j}$$

$$\frac{\partial r_2}{\partial \theta_1} = l_1 \cos\theta_1 \hat{\imath} - l_1 \sin\theta_1 \hat{j}$$

$$\Rightarrow \boxed{Q_1 = -(m_1 + m_2) g l_1 \sin\theta_1} \quad (2)$$

m_2 $F_1 = m_1 g \hat{j}$, $F_2 = m_2 g \hat{j}$

$$\frac{\partial r_1}{\partial \theta_2} = 0$$

$$\frac{\partial r_2}{\partial \theta_2} = l_2 \cos\theta_2 \hat{\imath} - l_2 \sin\theta_2 \hat{j}$$

$$\Rightarrow \boxed{Q_2 = -m_2 g l_2 \sin\theta_2} \quad (3)$$

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from equation (1) P1, we have the LHS of eq (1) for θ_1:

$$l_1\left[\ddot{\theta}_1(m_1 l_1 + m_2 l_1) + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1-\theta_2) - (\dot{\theta}_1 - \dot{\theta}_2) m_2 l_2 \dot{\theta}_2 \sin(\theta_1-\theta_2) + m_2 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_2)\right] = \underbrace{-(m_1+m_2) g l_1 \sin\theta_1}_{Q_1} \quad \Big/ \frac{1}{l_1}$$

$$\ddot{\theta}_1(m_1 l_1 + m_2 l_1) + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1-\theta_2) - (\dot{\theta}_1 - \dot{\theta}_2) m_2 l_2 \dot{\theta}_2 \sin(\theta_1-\theta_2) + m_2 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_2) = -(m_1+m_2) g \sin\theta_1$$

$$\Rightarrow \boxed{\ddot{\theta}_1 = \frac{-m_2 l_2 \left(\ddot{\theta}_2 \cos(\theta_1-\theta_2) + \dot{\theta}_2^2 \sin(\theta_1-\theta_2)\right) - (m_1+m_2) g \sin\theta_1}{l_1(m_1+m_2)}} \quad (4)$$

and from equation (3) P1, we have the LHS of eq (1) for θ_2:

$$m_2 l_2\left[l_2 \ddot{\theta}_2 + l_1 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - l_1 \dot{\theta}_1^2 \sin(\theta_1-\theta_2)\right] = -m_2 g l_2 \sin\theta_2 \quad \Big/ \frac{1}{m_2 l_2}$$

$$l_2 \ddot{\theta}_2 + l_1 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - l_1 \dot{\theta}_1^2 \sin(\theta_1-\theta_2) = -g \sin\theta_2$$

$$\Rightarrow \boxed{\ddot{\theta}_2 = \frac{l_1\left(\dot{\theta}_1^2 \sin(\theta_1-\theta_2) - \ddot{\theta}_1 \cos(\theta_1-\theta_2)\right) - g \sin\theta_2}{l_2}} \quad (5)$$

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Equations (4) and (5) depend upon each other and are thus recursive

Substituting (5) into (4) $\Rightarrow$ we get $\ddot{\theta}_1$

$$\boxed{\ddot{\theta}_1 = \left[-m_2\cos(\theta_1-\theta_2)l_1\dot{\theta}_1^2\sin(\theta_1-\theta_2) - m_2 l_2\dot{\theta}_2^2\sin(\theta_1-\theta_2) + m_2\cos(\theta_1-\theta_2)g\sin\theta_2 - (m_1+m_2)g\sin\theta_1\right] / \left[l_1(m_1+m_2) - m_2\cos^2(\theta_1-\theta_2)\right]} \quad (6)$$

Substituting (4) into (5) $\Rightarrow$ we get $\ddot{\theta}_2$

$$\boxed{\ddot{\theta}_2 = \left[(m_1+m_2)l_1\dot{\theta}_1^2\sin(\theta_1-\theta_2) + m_2 l_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)\cos(\theta_1-\theta_2) + (m_1+m_2)g\sin\theta_1\cos(\theta_1-\theta_2) - (m_1+m_2)g\sin\theta_2\right] / \left[l_2(m_1+m_2\sin^2(\theta_1-\theta_2)\right]} \quad (7)$$

Since $m_1 = m_2 = 2$, $l_1 = l_2 = 1$

$$\Rightarrow \boxed{\ddot{\theta}_1 = \frac{-\dot{\theta}_1^2\sin(\theta_1-\theta_2)\cos(\theta_1-\theta_2) - \dot{\theta}_2^2\sin(\theta_1-\theta_2) + \cos(\theta_1-\theta_2)g\sin\theta_2 - 2g\sin\theta_1}{2-\cos^2(\theta_1-\theta_2)}} \quad (8)$$

$$\Rightarrow \boxed{\ddot{\theta}_2 = \frac{2l_1\dot{\theta}_1^2\sin(\theta_1-\theta_2) + \dot{\theta}_2^2\sin(\theta_1-\theta_2)\cos(\theta_1-\theta_2) + 2g\sin\theta_1\cos(\theta_1-\theta_2) - 2g\sin\theta_2}{1+\sin^2(\theta_1-\theta_2)}} \quad (9)$$