7.8.7

$$\dot{x} = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix} x \qquad x(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\det \begin{bmatrix} 1-\lambda & -4 \\ 4 & -7-\lambda \end{bmatrix} = 0 = (1-\lambda)(-7-\lambda) + 16$$

$$0 = \lambda^2 + 7\lambda - \lambda + 16 - 7 = \lambda^2 + 6\lambda + 9$$

$$0 = (\lambda+3)^2 \qquad \lambda = -3, -3$$

$$(A - \lambda I)\xi = 0$$

$$\left[\begin{array}{cc|c} +4 & -4 & 0 \\ 4 & -4 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 4 & -4 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow 1 \text{ row of 0's, 1 reg. eigenvector}$$

$$\hat{\xi}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\left[\begin{array}{cc|c} 4 & -4 & 1 \\ 4 & -4 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -1 & 1/4 \\ 0 & 0 & 0 \end{array}\right] \quad x - y = 1/4 x$$

$$3/4 x = y \quad \Rightarrow \quad \xi_1^2 = \begin{bmatrix} 1 \\ 3/4 \end{bmatrix}$$

Gen Soln:

$$\xi_1(t) = c_1 \xi_1^1 e^{-3t} + c_2(\xi_1^2 + t\xi_1^1)e^{-3t}$$

$$\xi_1(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-3t} + c_2\left(\begin{bmatrix} 1 \\ 3/4 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)e^{-3t}$$

$$\xi(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} c_1 + c_2 \\ c_1 + 3/4 c_2 \end{bmatrix}$$

$$c_1 + c_2 = 3 \qquad c_1 = 3 - c_2$$

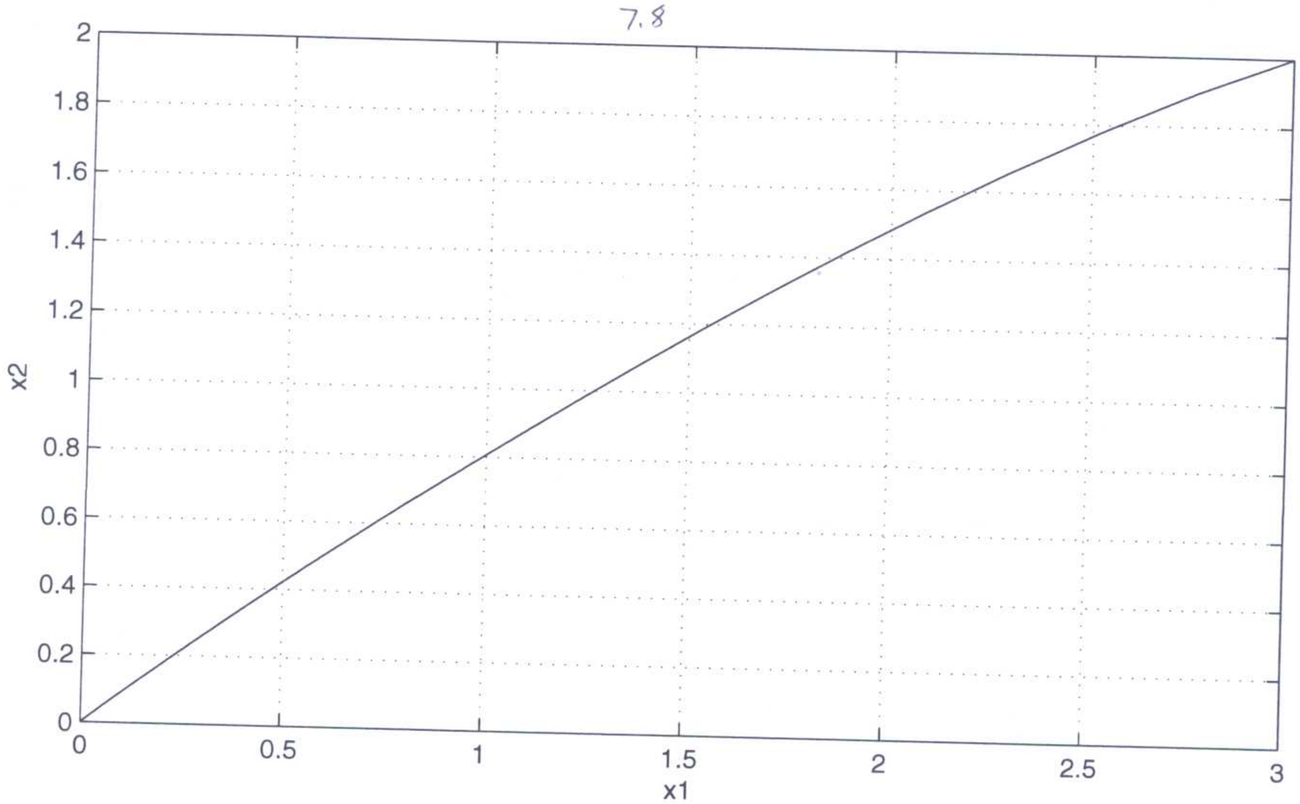
$$c_1 + 3/4 c_2 = 2 \quad \therefore \quad 3 - c_2 + 3/4 c_2 = 2$$

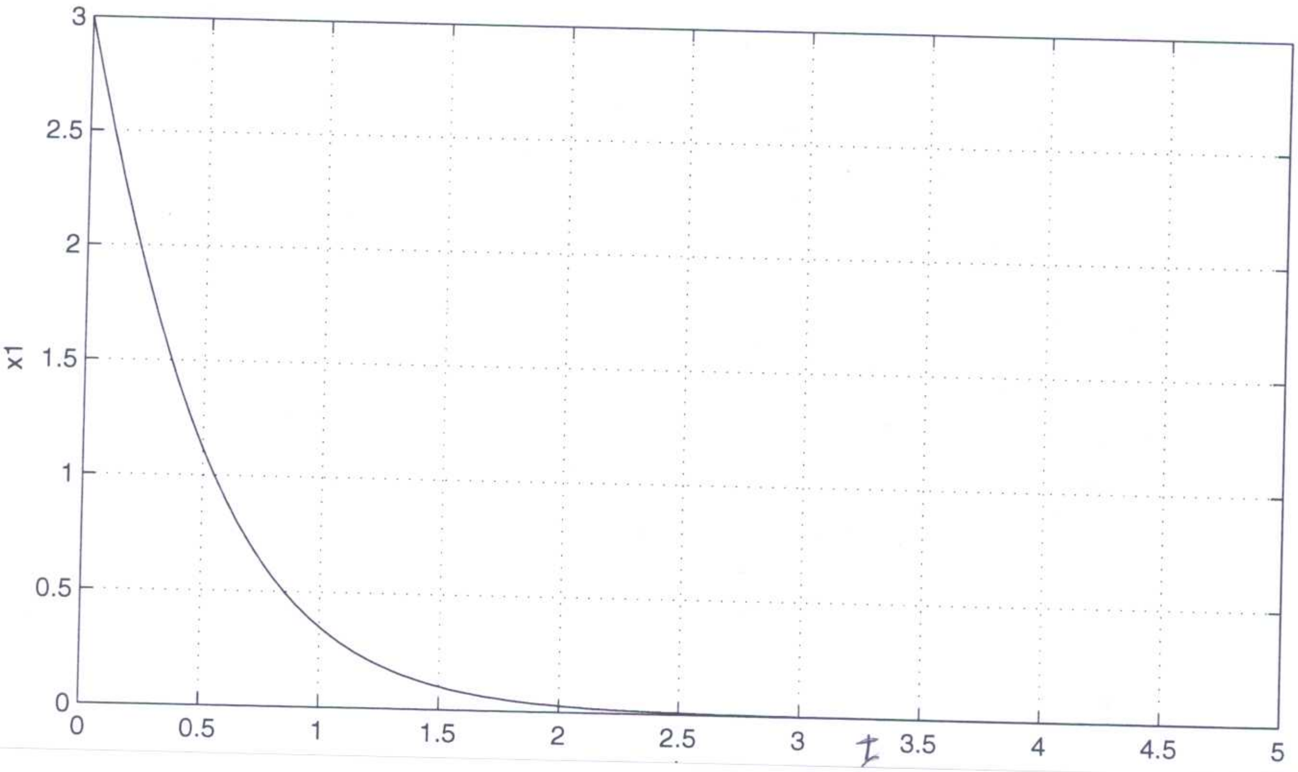
$$-\tfrac{1}{4} c_2 = -1 \qquad c_2 = 4 \qquad c_1 = -1$$

$$x(t) = -1\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-3t} + 4\left[\begin{bmatrix} 1 \\ 3/4 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right] e^{-3t}$$

$$x(t) = \begin{bmatrix} 3 + 4t \\ 2 + 4t \end{bmatrix} e^{-3t}$$

7.8





7.8.10

$$\dot{x} = \begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix} x \qquad x(0) = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$\det\begin{bmatrix} +3-\lambda & 9 \\ -1 & -3-\lambda \end{bmatrix} = 0 = (3-\lambda)(-3-\lambda) + 9 = 0$$

$$\lambda^2 + 3\lambda - 3\lambda - 9 + 9 = 0$$

$$\lambda^2 = 0 \quad \lambda = 0,0$$

$(A - \lambda I)\xi_1 = 0$

$$\left[\begin{array}{cc|c} 3 & 9 & 0 \\ -1 & -3 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 3 & 9 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad 3x + 9y = 0$$

$$3x = -9y \Rightarrow \xi_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

• one reg. eigenvector $\Rightarrow$ one gen. eigenvector.

$$\left[\begin{array}{cc|c} 3 & 9 & -3 \\ -1 & -3 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 3 & 9 & -3 \\ 0 & 0 & 0 \end{array}\right] \quad 3x + 9y = -3x$$

$$9y = -6x \Rightarrow \xi_1^{2} = \begin{bmatrix} 1 \\ -2/3 \end{bmatrix}$$

Gen Soln: $x(t) = c_1 \xi_1 e^{0t} + c_2[\xi_1^{2} + t\xi_1^{1}]e^{0t}$

$$= c_1\begin{bmatrix} -3 \\ 1 \end{bmatrix} + c_2\left[\begin{bmatrix} 1 \\ -2/3 \end{bmatrix} + t\begin{bmatrix} -3 \\ 1 \end{bmatrix}\right]$$

$$x(0) = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -3c_1 & +c_2 \\ c_1 & -2/3 c_2 \end{bmatrix} \Rightarrow 2 = -3c_1 + c_2 \Rightarrow c_2 = 2 + 3c_1$$

$$c_1 - \tfrac{2}{3}c_2 = 4$$

$$c_1 - \tfrac{2}{3}[2 + 3c_1] = 4$$

$$c_1 - \tfrac{4}{3} - 2c_1 = 4$$

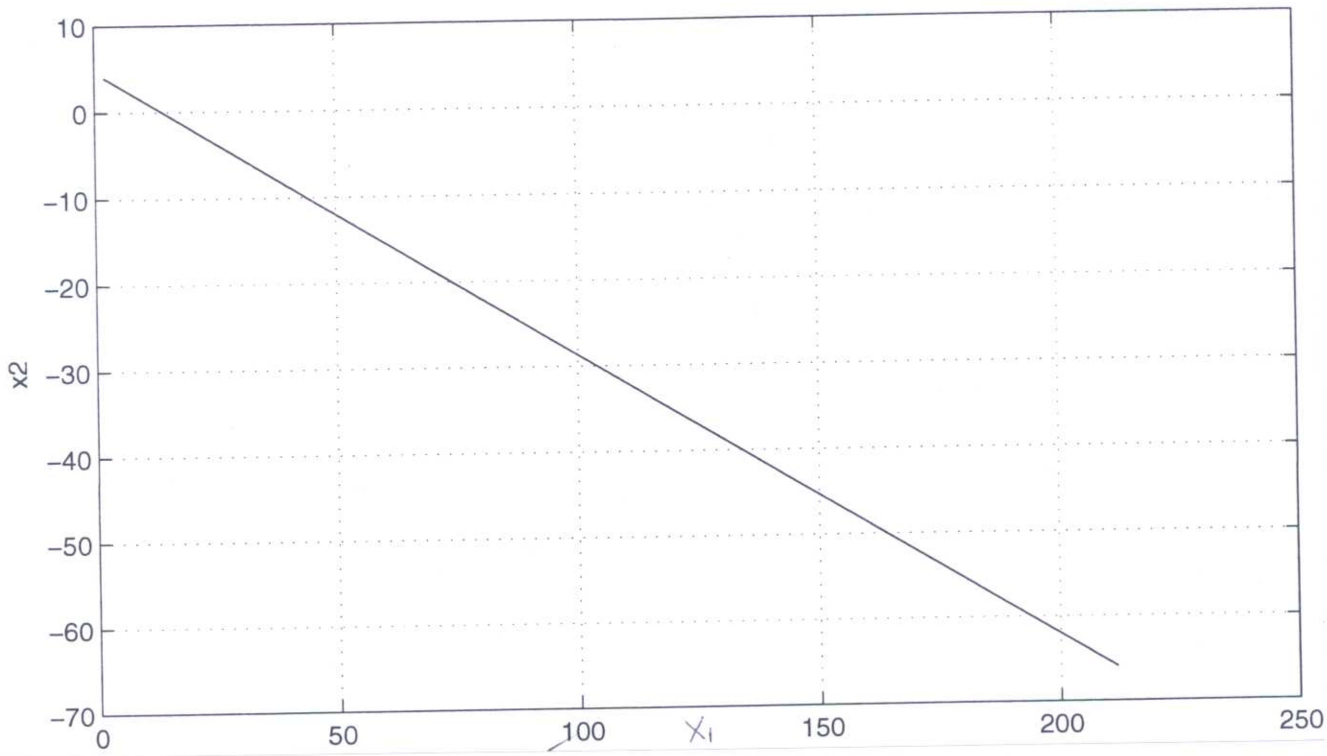
$$-c_1 = \tfrac{16}{3} \qquad c_1 = -\tfrac{16}{3}$$

$$c_2 = -14$$

$$x(t) = -\frac{16}{3}\begin{bmatrix} -3 \\ 1 \end{bmatrix} - 14\left[\begin{bmatrix} 1 \\ -2/3 \end{bmatrix} + t\begin{bmatrix} -3 \\ 1 \end{bmatrix}\right]$$

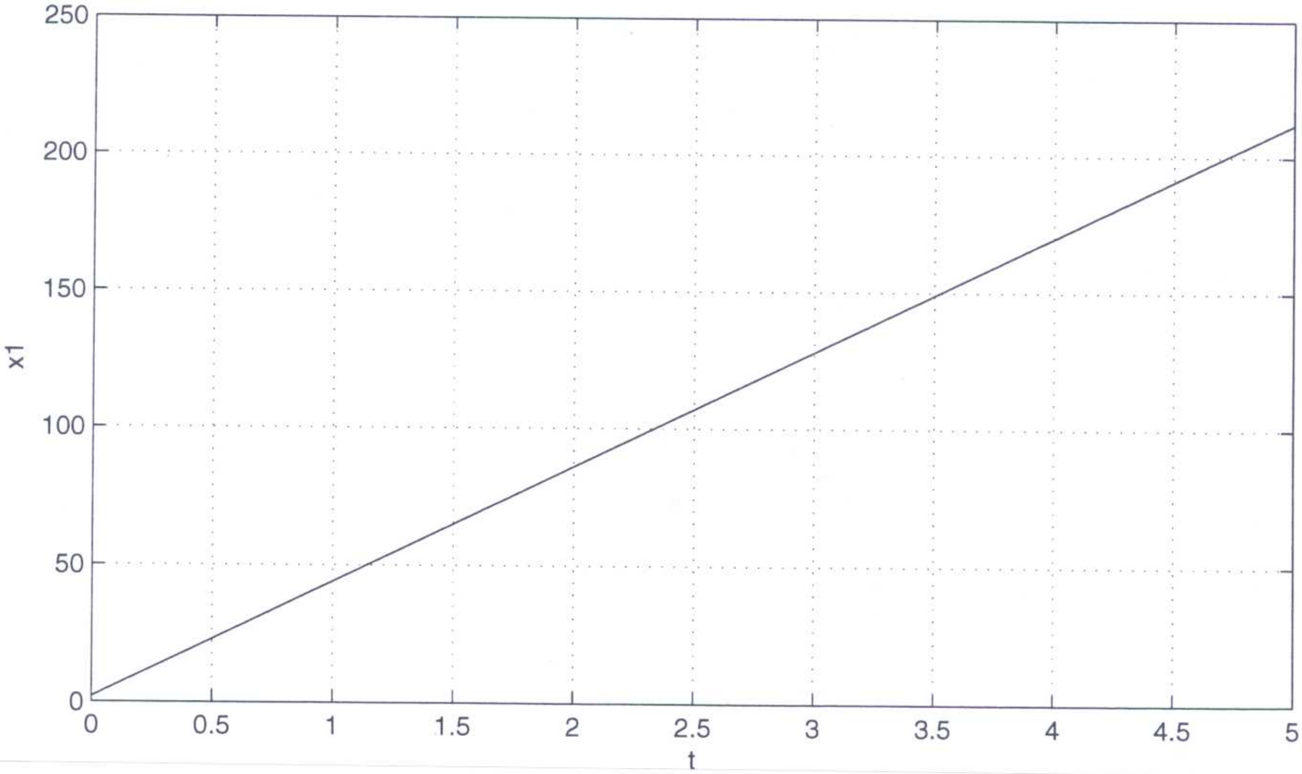
$$x(t) = \begin{bmatrix} 2 \\ 4 \end{bmatrix} - 14\begin{bmatrix} -3 \\ 1 \end{bmatrix} t$$

7.10



10
0
−10
−20
−30
−40
−50
−60
−70
x2
0
50
100
150
200
250
X_1

7.10



250
200
150
100
50
0
x1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t

Problem 2

$\dot{\xi} = A\xi$

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

- Note, since A is upper triangular, we can read λ's off of the diagonal. In our case, $\lambda = 2, 2, 2, 2, 2$

- Look for regular eigenv's.

$(A - \lambda_1 I)\xi_1^1 = 0$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

1 row of zeros $\Rightarrow$ 1 reg. eigenvecter

$$\xi_1^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- Generalized ev's

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \xi_1^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \xi_1^3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \xi_1^4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad \left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \xi_1^5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

General Solution:

$$\xi(t) = c_1 \xi_1^1 e^{2t} + c_2(\xi_1^2 + \xi_1^1 t)e^{2t} + c_3\left(\xi_1^3 + t\xi_1^2 + \frac{t^2}{2}\xi_1^1\right)e^{2t}$$

$$+ c_4\left(\xi_1^4 + t\xi_1^3 + \frac{t^2}{2}\xi_1^2 + \frac{t^3}{3}\xi_1^1\right)e^{2t}$$

$$+ c_5\left(\xi_1^5 + t\xi_1^4 + \frac{t^2}{2}\xi_1^3 + \frac{t^3}{3}\xi_1^2 + \frac{t^4}{4}\xi_1^1\right)e^{2t}$$

Problem 3

$\dot{\xi} = A\xi \qquad A = \begin{bmatrix} 4 & 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

- Since A is upper triangular we read λ's off of the diagonal.
  $\lambda = 4, 4, 2, 2, 2$

- Find ev's for $\lambda = 4$
  - ↳ Regular: $\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \end{array}\right]$ ⇒ one row of zeros, so one reg. ev
    $\xi_1^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
  - ↳ Generalized $\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \end{array}\right]$ ⇒ $\xi_1^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

- Find ev's for $\lambda = 2$
  - ↳ Regular: $\left[\begin{array}{ccccc|c} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ ⇒ One row of zeros, so one reg. ev
    $\xi_2^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$
  - ↳ Generalized
    $\left[\begin{array}{ccccc|c} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ ⇒ $\xi_2^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $\left[\begin{array}{ccccc|c} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ ⇒ $\xi_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

- General Solution

$$\xi(t) = c_1 \xi_1^1 e^{4t} + c_2(\xi_1^2 + \xi_1^1 t)e^{4t} + c_3 \xi_2^1 e^{2t} + c_4(\xi_2^2 + \xi_2^1 t)e^{2t}$$

$$+ c_5\left(\xi_2^3 + \xi_2^2 t + \frac{t^2}{2}\xi_2^1\right)e^{2t}$$

Problem 4

$\dot{\xi} = A\xi \qquad A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

- Since A is triangular $\lambda = 2,2,2,2,2$

- Find ev's for $\lambda = 2$
  - ↳ Regular

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

2 Rows of 0's
$\Rightarrow$ 2 regular ev's

$\xi_1^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \xi_2^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

  - ↳ Generalized ξ_1^1

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \text{no solution}$$

  - ↳ ξ_2^1

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \xi_2^2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Similarly $\xi_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \xi_2^4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

- General Solution

$$\xi(t) = c_1 \xi_1^1 e^{2t} + c_2 \xi_2^1 e^{2t} + c_3 (\xi_2^1 t + \xi_2^2) e^{2t}$$

$$+ c_4 (\xi_2^1 t^2/2 + \xi_2^2 t + \xi_2^3) e^{2t} + c_5 (t^3/3\, \xi_2^1 + \tfrac{t^2}{2} \xi_2^2$$

$$+ t \xi_2^3 + \xi_2^4) e^{2t}$$

Problem 5

$\dot{\xi} = \xi A \qquad A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

- Since A is triangular $\lambda = 2, 2, 2, 2, 2$

- Find ev's for $\lambda = 2$

↳ Regular : $\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ → 3 rows of zeros gives 3 regular ev's

$\xi_1^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \xi_2^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \xi_3^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

↳ Generalized

$\xi_1^1 : \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ → no solution

$\xi_2^1 : \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ → no solution

$\xi_3^1 : \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \xi_3^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

Similarly, $\xi_3^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

General Solution:

$$\xi(t) = c_1 \xi_1^1 e^{2t} + c_2 \xi_2^1 e^{2t} + c_3 \xi_3^1 e^{2t}$$

$$+ c_4 \left(t \xi_3^1 + \xi_3^2 \right) e^{2t} + c_5 \left(\tfrac{t^2}{2} \xi_3^1 + t \xi_3^2 + \xi_3^3 \right) e^{2t}$$

Problem 6

$\dot{\xi} = A\xi$ $\qquad A = \begin{bmatrix} -4 & 1 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$

- Since A is triangular $\lambda = -4, -4, -4, -2, -2$

- Find regular ev for $\lambda = -4$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{array}\right] \Rightarrow \text{2 rows of 0 give two reg. ev's}$$

$$\xi_1^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \xi_2^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

↳ Generalized ξ_1^1:

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{array}\right] \Rightarrow \xi_1^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

ξ_2^1

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{array}\right] \Rightarrow \text{no solution}$$

- Find ev's for $\lambda = -2$

$$\left[\begin{array}{ccccc|c} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \text{1 row of 0's} \Rightarrow \text{1 reg ev}$$

$$\xi_3^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

↳ Generalized:

$$\left[\begin{array}{ccccc|c} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \xi_3^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

- Gen Soln:

$$\xi(t) = c_1 \xi_1^1 e^{-4t} + c_2 \xi_2^1 e^{-4t} + c_3 (t\xi_1^1 + \xi_1^2) e^{-4t}$$

$$+ c_4 \xi_3^1 e^{-2t} + c_5 (\xi_3^1 t + \xi_3^2) e^{-2t}$$

Problem 7

- Find the exact solution $\quad \xi(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

- From (6)

$$\xi(0) = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow c_1 = c_2 = c_3 = c_4 = c_5 = 1$$

$$\xi(t) = \xi_1^1 e^{-4t} + \xi_2^1 e^{-4t} + (t\xi_1^1 + \xi_1^2)e^{-4t} + \xi_3^1 e^{-2t} + (\xi_3^1 t + \xi_3^2)e^{-4t}$$