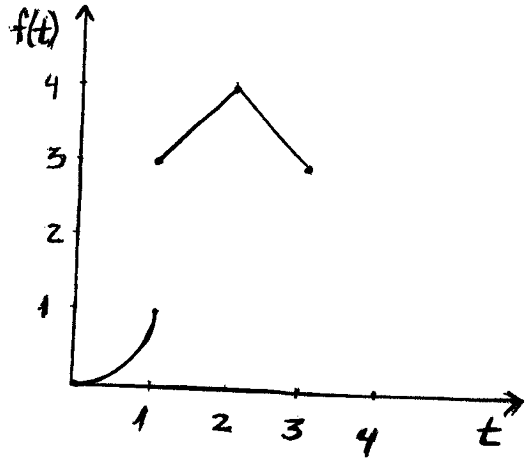
P1)

6.1.1

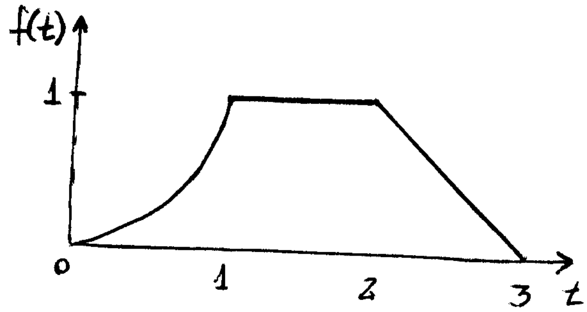
$$f(t) = \begin{cases} t^2 & 0 \le t \le 1 \\ 2+t & 1 < t \le 2 \\ 6-t & 2 < t \le 3 \end{cases}$$



$f(t)$ is piecewise continuous on the interval $0 \le t \le 3$

6.1.3

$$f(t) = \begin{cases} t^2 & 0 \le t \le 1 \\ 1 & 1 < t \le 2 \\ 3-t & 2 < t \le 3 \end{cases}$$



$f(t)$ is continuous

6.1.7

$$\mathcal{L}(\cosh bt) = \int_0^\infty e^{-st} \cosh bt\, dt = \int_0^\infty e^{-st} \cdot \frac{(e^{bt} + e^{-bt})}{2}\, dt$$

$$= \frac{1}{2}\left[\int_0^\infty e^{(b-s)t} dt + \int_0^\infty e^{-(b+s)t} dt\right] = \frac{1}{2}\int_0^\infty e^{-(s-b)t} dt + \frac{1}{2}\int_0^\infty e^{-(s+b)t} dt$$

$$= \frac{1}{2}\left(\frac{1}{s-b}\right) + \frac{1}{2}\left(\frac{1}{s+b}\right) \Rightarrow \mathcal{L}(\cosh bt) = \frac{1}{2}\left(\frac{2s}{s^2-b^2}\right)$$

$$\Rightarrow \boxed{\mathcal{L}(\cosh bt) = \frac{s}{s^2-b^2}}, \quad s > |b|$$

P2

6.2.11

$y'' - y' - 6y = 0 \; / \mathcal{L} \qquad y(0) = 1, \quad y'(0) = -1$

$\mathcal{L}(y'') - \mathcal{L}(y') - \mathcal{L}(6y) = 0$

$s^2\mathcal{L}(y) - sy(0) - y'(0) - [s\mathcal{L}(y) - y(0)] - 6\mathcal{L}(y) = 0$

$s^2\mathcal{L}(y) - s + 1 - s\mathcal{L}(y) + 1 - 6\mathcal{L}(y) = 0$

$(s^2 - s - 6)\mathcal{L}(y) = s - 2$

$\downarrow$

$Y(s) = \dfrac{s-2}{s^2-s-6} \Rightarrow Y(s) = \dfrac{s-2}{(s+2)(s-3)}$

$Y(s) = \dfrac{a}{(s+2)} + \dfrac{b}{(s-3)}, \qquad a(s-3) + b(s+2) = s-2$

$(a+b)s - 3a + 2b = s - 2$

$a + b = 1, \quad -3a + 2b = -2$

$a = 1 - b \Rightarrow -3(1-b) + 2b = -2$

$-3 + 3b + 2b = -2$

$5b = 1$

$b = \frac{1}{5}, \quad a = \frac{4}{5}$

$\Rightarrow Y(s) = \dfrac{4/5}{s+2} + \dfrac{1/5}{s-3}$

$\Rightarrow \boxed{y(t) = \frac{4}{5}e^{-2t} + \frac{1}{5}e^{3t}}$

P2)

6.2.15

$$y'' - 2y' + 4y = 0 \quad / \mathcal{L} \qquad y(0) = 2, \; y'(0) = 0$$

$$\mathcal{L}(y'') - \mathcal{L}(2y') + \mathcal{L}(4y) = 0$$

$$s^2 \mathcal{L}(y) - s y(0) - y'(0) - 2\left[s\mathcal{L}(y) - y(0)\right] + 4\mathcal{L}(y) = 0$$

$$s^2 \mathcal{L}(y) - 2s - 0 - 2s\mathcal{L}(y) + 4 + 4\mathcal{L}(y) = 0$$

$$(s^2 - 2s + 4)\mathcal{L}(y) = 2s - 4$$

$$\downarrow$$

$$Y(s) = \frac{2s-4}{(s^2-2s+4)} = \frac{2(s-2)}{(s-1)^2 + (\sqrt{3})^2}$$

$$Y(s) = \frac{2(s-1)}{(s-1)^2 + (\sqrt{3})^2} - \frac{2}{(s-1)^2 + (\sqrt{3})^2}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$\boxed{y(t) = 2e^t \cos\sqrt{3}t - \frac{2}{\sqrt{3}} e^t \sin\sqrt{3}t}$$

P2.

6.2.22

$$y'' - 2y' + 2y = e^{-t} \quad / \mathcal{L} \qquad y(0) = 0, \; y'(0) = 1$$

$$\mathcal{L}(y'') - \mathcal{L}(2y') + \mathcal{L}(2y) = \mathcal{L}(e^{-t})$$

$$s^2 \mathcal{L}(y) - s y(0) - y'(0) - 2[s\mathcal{L}(y) - y(0)] + 2\mathcal{L}(y) = \mathcal{L}(e^{-t})$$

$$s^2 \mathcal{L}(y) - 0 - 1 - 2s\mathcal{L}(y) + 2\mathcal{L}(y) = \frac{1}{s+1}$$

$$(s^2 - 2s + 2)\mathcal{L}(y) = \frac{1}{s+1} + 1$$

$$\downarrow$$

$$Y(s) = \frac{s+2}{(s+1)(s^2-2s+2)}$$

$$Y(s) = \frac{a}{(s+1)} + \frac{bs+c}{(s^2-2s+2)}$$

$$a(s^2-2s+2) + (bs+c)(s+1) = s+2$$

$$as^2 - 2as + 2a + bs^2 + bs + cs + c = s+2$$

$$(a+b)s^2 + (b+c-2a)s + 2a + c = s+2$$

$$a+b=0 \Rightarrow a=-b$$

$$\left.\begin{array}{l} b+c-2a = 1 \Rightarrow 3b+c=1 \\ 2a+c = 2 \Rightarrow -2b+c=2 \end{array}\right\} \Rightarrow 5b = -1 \Rightarrow b = -\frac{1}{5}$$

$$a = \frac{1}{5}$$

$$c = 2 - \frac{2}{5} \Rightarrow c = \frac{8}{5}$$

P2

6.2.22 (Cont.)

$$Y(s) = \frac{1/5}{s+1} + \frac{-1/5\, s + 8/5}{s^2-2s+2}$$

$$Y(s) = \frac{1/5}{s+1} + \frac{1}{5}\left[\frac{-s+8}{s^2-2s+2}\right]$$

$$Y(s) = \frac{1}{5}\left(\frac{1}{s+1} - \frac{s-1}{s^2-2s+2} + \frac{7}{s^2-2s+2}\right)$$

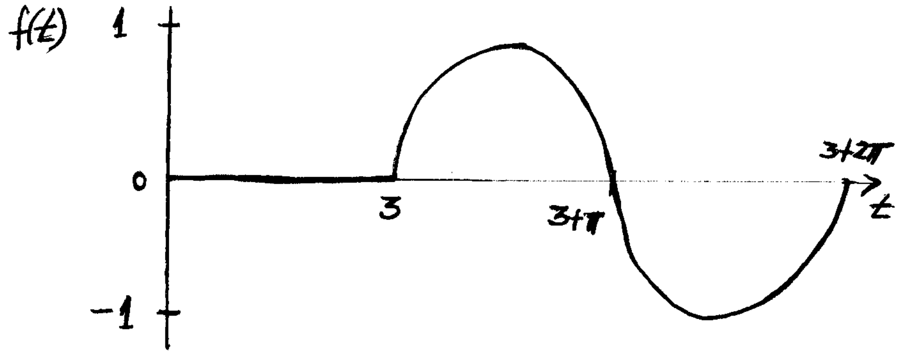
$$Y(s) = \frac{1}{5}\left(\frac{1}{s+1} - \frac{s-1}{(s-1)+(1)^2} + \frac{7}{(s-1)^2+(1)^2}\right)$$

$$\boxed{y(t) = \frac{1}{5}e^{-t} - \frac{1}{5}e^{t}\cos t + \frac{7}{5}e^{t}\sin t}$$

P3

6.3.4

$f(t-3)\,u_3(t)$, where $f(t) = \sin t$



6.3.8

$$f(t) = \begin{cases} 0 & t < 1 \\ t^2-2t+2 & t \geq 1 \end{cases}$$

$$f(t) = \begin{cases} 0 & t < 1 \\ (t-1)^2+1 & t \geq 1 \end{cases}$$

$$f(t) = u_1(t)\left[(t-1)^2+1\right]$$

$$\mathcal{L}(f(t)) = \mathcal{L}\left(u_1(t)\left[(t-1)^2+1\right]\right)$$

$$\mathcal{L}(f(t)) = e^{-s}\mathcal{L}(t^2+1) = e^{-s}\left(\frac{2}{s^3}+\frac{1}{s}\right)$$

$$\boxed{\mathcal{L}(f(t)) = \left(\frac{2}{s^3}+\frac{1}{s}\right)e^{-s}}$$

P3

6.3.9

$$f(t) = \begin{cases} 0 & t < \pi \\ t-\pi & \pi \le t < 2\pi \\ 0 & t \geq 2\pi \end{cases}$$

$$\Rightarrow f(t) = u_{\pi}(t)(t-\pi) - u_{2\pi}(t)(t-\pi)$$

$$f(t) = u_{\pi}(t)(t-\pi) - u_{2\pi}(t)(t-2\pi+\pi)$$

$$\mathcal{L}(f(t)) = \mathcal{L}(u_{\pi}(t)(t-\pi)) - \mathcal{L}(u_{2\pi}(t)(t-2\pi+\pi))$$

$$\mathcal{L}(f(t)) = e^{-\pi s}\mathcal{L}(t) - e^{-2\pi s}\mathcal{L}(t+\pi)$$

$$\boxed{\mathcal{L}(f(t)) = e^{-\pi s}\left(\frac{1}{s^2}\right) - e^{-2\pi s}\left(\frac{1}{s^2}+\frac{\pi}{s}\right)}$$

P4

6.4.1

$$y'' + y = f(t) \qquad y(0) = 0, \quad y'(0) = 1$$

$$f(t) = \begin{cases} 1 & 0 \leq t < \pi/2 \\ 0 & \pi/2 \leq t < \infty \end{cases}$$

$$y'' + y = f(t) \quad / \mathcal{L}$$

$$\mathcal{L}(y'') + \mathcal{L}(y) = \mathcal{L}(f(t))$$

$$s^2 \mathcal{L}(y) - s\,y(0) - y'(0) + \mathcal{L}(y) = \mathcal{L}(f(t))$$

$$s^2 \mathcal{L}(y) - 1 + \mathcal{L}(y) = \mathcal{L}(f(t))$$

$$f(t) = 1 - u_{\pi/2}(t)$$

$$\mathcal{L}(f(t)) = \frac{1}{s} - e^{-\frac{\pi}{2}s} \cdot \frac{1}{s} = \frac{1}{s}\left(1 - e^{-\frac{\pi}{2}s}\right)$$

$$\Rightarrow (s^2+1)\mathcal{L}(y) = \frac{1}{s}\left(1 - e^{-\frac{\pi}{2}s}\right) + 1$$

$$\downarrow$$

$$Y(s) = \frac{(1 - e^{-\frac{\pi}{2}s})}{s(s^2+1)} + \frac{1}{(s^2+1)}$$

$$Y(s) = \left(1 - e^{-\frac{\pi}{2}s}\right) H(s) + \frac{1}{s^2+1}$$

$$y(t) = h(t) - u_{\pi/2}(t) h(t - \pi/2) + \sin t \quad , \quad h(t) = \mathcal{L}^{-1}(H(s))$$

$$H(s) = \frac{1}{s(s^2+1)} = \frac{a}{s} + \frac{bs+c}{s^2+1}$$

$$a(s^2+1) + bs^2 + cs = 1$$

$$as^2 + a + bs^2 + cs = 1 \Rightarrow \begin{array}{l} c = 0 \\ a = 1 \\ b = -1 \end{array}$$

P4

6.4.1 (cont.)

$$H(s) = \frac{1}{s} - \frac{s}{s^2+1}$$

$$\Rightarrow h(t) = 1 - \cos t$$

$$\Rightarrow \boxed{y(t) = 1 - \cos(t) - u_{\pi/2}(t) \cdot \left(1 - \cos\left(t - \tfrac{\pi}{2}\right)\right) + \sin t}$$

or

$$\boxed{y(t) = u_0(t) \cdot (1 - \cos(t)) - u_{\pi/2}(t)\left(1 - \cos\left(t - \tfrac{\pi}{2}\right)\right) + \sin t}$$

P4

6.4.9

$$y'' + y = g(t) \qquad y(0)=0,\ y'(0)=1$$

$$g(t) = \begin{cases} t/2, & 0 \le t < 6 \\ 3, & t \geq 6 \end{cases}$$

$$y'' + y = g(t) \quad / \mathcal{L}$$

$$\mathcal{L}(y'') + \mathcal{L}(y) = \mathcal{L}(g(t))$$

$$s^2 \mathcal{L}(y) - s\,y(0) - y'(0) + \mathcal{L}(y) = \mathcal{L}(g(t))$$

$$s^2 \mathcal{L}(y) - 1 + \mathcal{L}(y) = \mathcal{L}(g(t))$$

$$g(t) = \left[u_0(t)(t) - u_6(t)(t-6)\right]/2$$

$$\mathcal{L}(g(t)) = \frac{1}{2s^2} - \frac{1e^{-6s}}{2s^2} = \frac{1}{2s^2}\left(1-e^{-6s}\right)$$

$$(s^2+1)\mathcal{L}(y) = \frac{1}{2s^2}\left(1-e^{-6s}\right) + 1$$

$$\downarrow$$

$$Y(s) = \frac{(1-e^{-6s})}{2s^2(s^2+1)} + \frac{1}{(s^2+1)}$$

$$Y(s) = (1-e^{-6s})H(s) + \frac{1}{s^2+1}$$

$$y(t) = h(t) - u_6(t)\,h(t-6) + \sin t \quad , \quad h(t) = \mathcal{L}^{-1}(H(s))$$

$$H(s) = \frac{1}{2s^2(s^2+1)} = \frac{1}{2}\left[\frac{a}{s^2} + \frac{b}{s^2+1}\right]$$

$$as^2 + a + bs^2 = 1 \Rightarrow \begin{matrix} a = 1 \\ b = -1 \end{matrix}$$

P4

6.4.9 (Cont.)

$$H(s) = \frac{1}{2}\left[\frac{1}{s^2} - \frac{1}{s^2+1}\right]$$

$$h(t) = \frac{1}{2}t - \frac{1}{2}\sin t$$

$$\Rightarrow y(t) = \frac{1}{2}t - \frac{1}{2}\sin t - u_6(t)\left(\frac{1}{2}(t-6) - \frac{1}{2}\sin(t-6)\right) + \sin t$$

$$\boxed{y(t) = \frac{1}{2}t + \frac{1}{2}\sin t - u_6(t)\left(\frac{1}{2}t - 3 - \frac{1}{2}\sin(t-6)\right)}$$