University of Notre Dame
Aerospace and Mechanical Engineering

AME 34314: Differential Equations, Vibrations and Controls I
Exam 1

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NAME: ______________________________

- Do not start or turn the page until instructed to do so.
- You have 75 minutes to complete this exam.
- This is an open book exam. You may consult the course texts, any other text book, your class notes, homework solutions and your own homework sets.
- You may **not** use a calculator.
- There are five problems. Each problem is worth 20 points.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

Q - "Behind the headlights: Stinger missiles."
James Bond - "Excellent."

— *GoldenEye*

1. It is now 2008 and you are working for Q Branch in the British Secret Service. To aid James Bond in the fight against the Special Executive for Counterintelligence, Terrorism, Revenge and Extortion (SPECTRE), Q is developing a wristwatch that contains a miniature howitzer.

   The deployment mechanism is described by the following differential equation:

$$\ddot{x} + 4\dot{x} + 4x = e^{-2t}.$$

   Determine the general solution to this differential equation.

2. You have been asked to add additional functionality to the standard issue Aston Martin DB5 in the form of a high-speed titanium CD player ejector weapon.[1] At a critical stage of the design process, Q asks you to determine the solution to

$$\begin{aligned} \dot{x} + 2tx &= e^{-t^2} \\ x(0) &= 1. \end{aligned}$$

[1]This is in addition to the usual bulletproof front and rear panels, oil slick and smoke screen countermeasures, machine guns, rotating license plates, telescoping tire slashers, tracer receiving console, passenger ejector seat, lasers, adaptive camouflage, depth charges, submarine conversion capability, self-destruct mechanism, thermal imaging monitor, forward-looking infrared radar (FLIR), mortar bombs, gatling gun and radio jamming equipment.

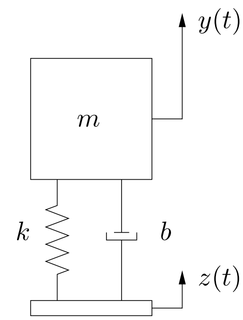

Figure 1. Gun mechanism for Problem 3.

3. In order to improve its aiming capabilities, you have been assigned to a team to redesign the gatling gun mechanism on the Aston Martin. A schematic of the mounting system is illustrated in Figure 1, where the block represents the gun, the spring and damper represent the mounting suspension mechanism and the small base represents the mounting plate mounted to the car frame. It is the case that typically, $z(t) = \cos \omega t$.

   You saved your notes from your undergraduate differential equations course, so you know that

$$m\ddot{y} + b\dot{y} + ky = k \cos \omega t - b \sin \omega t.$$

   The current design has

$$\begin{aligned} m &= 1 \\ b &= 2 \\ k &= 4, \end{aligned}$$

   and under standard operating conditions, $\omega \approx 1$.

   Your current options to modify the device are limited to changing the damper. One option increases the damping so that $b = 4$ and the other decreases it so that $b = 1$.

   If the goal is to keep the magnitude of the steady state response, $y_{ss}(t)$, small, which is the best option: increasing b, decreasing b or keeping it the same?

   Justify your answer by solving all or part or the differential equation and analyzing the solution.

4. Regardless of your answer to the previous problem, budget cuts have prevented implementing any design changes to the gatling gun on the Aston Martin, and in fact, in a cost savings measure, the damper was removed. Hence, the displacement of the gun is given by

$$m\ddot{x} + kx = k\cos\omega t$$

where

$$\begin{aligned} m &= 1 \\ k &= 1. \end{aligned}$$

The frequency at which the car shakes, ω, is proportional to the speed of the car. Q tells you that if s is the speed of the car in miles per hour, then

$$\omega = \frac{s}{60}.$$

Since it is easiest to aim the gun when the magnitude of the steady state response, $y_{ss}(t)$ is small, what advice would you give the Bond regarding the speed to drive? Justify your answer mathematically.

5. James Bond likes to impress enemy agents with his suave sophistication and knowledge of differential equations. To help him with the latter, determine the following.

   Consider the ordinary, second order, constant coefficient, linear, homogeneous differential equation

   $$a\ddot{x} + b\dot{x} + cx = 0.$$

   (a) Is it true that if a, b and c are all greater than zero, then

   $$\lim_{t\to\infty} x(t) = 0?$$

   (b) Is it true that if any of a, b and c are less than zero, then the $x(t)$ becomes unbounded?

   Justify your answer mathematically.