University of Notre Dame
Aerospace and Mechanical Engineering

AME 301: Differential Equations, Vibrations and Controls
Exam 2

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NAME: ______________________________

- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, your class notes, your own homework sets and any documents provided on the course homepage such as homework solutions, *etc.* You may **not** use a calculator, except on problem 1.
- There are five questions. Problems 1, 2 and 4 are each worth 20 points, problem 3 is worth 30 points and problem 5 is worth 10 points.
- If a problem requires to you solve a differential equation, you *may* use the convolution integral, but if you choose to do so, you must evaluate the integral, unless otherwise indicated.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page or on the blank pages. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- Do not start or turn the page until instructed to do so.

Oysters are supposed to be the first course. Or terrapin. Then the soup, then the fish, then the mushrooms or asparagus, then the roast, then the frozen punch, then the game, then the salad, then the creamed dessert, then the frozen dessert, then the cheese, then the fruit, and then hungry guests can get into the candy and nuts.

— Judith Martin (Miss Manners), August 21, 1996.

1. A spring mass system is subjected to Coulomb damping, as is illustrated in Figure 1.

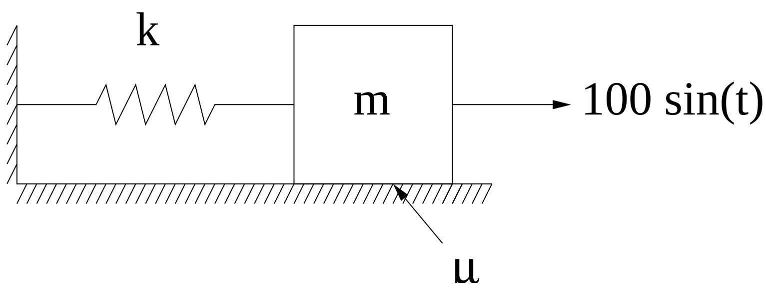

Figure 1. Mass spring system for Problem 1.

When a harmonic force of amplitude 100N and frequency of 1rad/sec is applied, the system is found to oscillate (at steady-state) with an amplitude of 1m. Determine an approximate coefficient of dry friction if m=2kg and k=100N/m. (20 points)

2. Solve

$$\ddot{x} + 2\dot{x} + 2x = 0 \qquad x(0) = 1 \qquad \dot{x}(0) = -1,$$

using the Laplace transform method (you can feel free to verify your answer using other techniques, if you desire). (20 points)

3. Solve

$$\ddot{x} + \dot{x} + x = g(t); \qquad x(0) = 0, \qquad \dot{x}(0) = 0; \qquad g(t) = \begin{cases} \sin t & 0 \le t < \pi \\ 2\sin t & t \ge \pi \end{cases}$$

using whatever method you choose. (30 points)

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4. Solve

$$\ddot{x} + 3\dot{x} + 2x = \delta(t-2) \qquad x(0) = 0 \qquad \dot{x}(0) = 1$$

using whatever method you choose. (20 points)

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5. A friend tells you the following:

The Laplace transform of the derivative of a function is s times the Laplace transform of the original function. We do this all the time when we take the Laplace transform of a differential equation:

$$\ddot{y}(t) + \dot{y}(t) + y(t) = 0 \Longleftrightarrow Y(s)(s^2 + s + 1) = 0.$$

So, consider $\cos(t)$. Obviously, $\frac{d}{dt}\cos(t) = -\sin(t)$. However, using the table, we see that

$$\mathfrak{L}\left[\frac{d}{dt}\cos(t)\right] = s\mathfrak{L}\left[\cos(t)\right] = s\frac{s}{s^2+1} = \frac{s^2}{s^2+1}$$

But looking at the table, we have

$$\mathfrak{L}\left[-\sin(t)\right] = -\frac{1}{s^2+1},$$

which is not the same thing.

Explain what was wrong about your friend's reasoning. (10 points)