

University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Fall 2015

Homework 8, due November 4th, 2015

Problem(1) Consider $\ddot{x} - 2t\dot{x} + 0.2x = 0$.

1. Is this one of the differential equations outlined in Section 5.5 in the textbook?
2. Is the point $t = 0$ an ordinary point or a singular point?
3. Assume a series solution and substitute to determine the coefficients.
4. If $x(0) = 1$, $\dot{x}(0) = 1$ and plot the solution for various numbers of terms. Compare your series solutions to a solution obtained numerically.

Problem(2) Determine a partial sum of the series solution about $t = 0$ to

$$\ddot{x} + 5(t^2 + 1)\dot{x} + x = 0,$$

where $x(0) = 1$ and $\ddot{x}(0) = 1$.

Compare your partial sum approximations to a solution determined numerically.

Problem(3) Determine a series solution to Airy's equation

$$\frac{d^2y}{dx^2} + xy = 0,$$

where $y(0) = 1$ and $dy/dx(0) = -1$.

And, compare your partial sum approximations (up to $n = 4$) to a solution determined numerically.

Problem(4) For the Hermite equation:

1. Determine a series solution about $t = 0$ when $\lambda = 3$ and $y(0) = 1$ and $y'(0) = 0$.
2. Determine a series solution about $t = 0$ when $\lambda = 4$ and $y(0) = 0$ and $y'(0) = 1$.

For each case, plot your solution and compare it to an accurate approximate numerical solution.

Problem(5) Consider

$$\ddot{x} + \frac{1}{t^2 + 2t + 2}\dot{x} + \frac{1}{t^2 + 4t + 8}x = 0,$$

1. (only required) For what interval of t would a series solution about $t = 0$ converge?
2. (extra credit) Determine the recursion relation for the series solution.
3. (extra credit) Compare the series solution for partial sums including different numbers of terms for the following two cases.
 - i. $x(0) = 1$ and $\dot{x}(0) = -1$.
 - ii. $x(0) = 5$ and $\dot{x}(0) = 0$. Do the initial conditions seem to affect the interval of convergence?

Note that For the problems that say to compare it with a numerical solution. It is permissible to use `ode45()`. You may write your own code as well. Doing so may pay off in a few weeks when you will be required to do it.