University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Second Exam

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- Do not start or turn the page until instructed to do so. You have 75 minutes to complete this exam.
- This is an open book exam. You may consult the course text and your own course notes, but nothing else.
- There are five problems, each worth 20 points.
- Unless stated otherwise, assume that a spring is unstretched when $x = 0$.
- You may **not** use a calculator or other computational aid.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in a Blue Book.
- You may choose whatever method you like to solve the problems unless the problem specifies which method to use. Merely substituting into an equation from the book is totally fine as long as it answers the problem.

I'm not absolutely certain of the facts, but I rather fancy it's Shakespeare who says that it's always just when a fellow is feeling particularly braced with things in general that Fate sneaks up behind him with the bit of lead piping.

— P.G. Wodehouse, *Carry on, Jeeves*

1. It is now 2020 and you work for Exxon Mobil on a offshore platform project.[1] You need to design your structure so that the waves do not rock it too much. A schematic of the *compliant tower* model they are considering is illustrated in Figure 1.

   Your model is like that from Exercise 4.26 in the course text, where, as the platform is pushed sideways by $f_w(t)$, the force of the waves, and the long slender part of the structure deflects to the left or right by an amount x, the restoring force is given by $f = (3EI/L^3)x$. Assume that

   - $L = 1000[m]$
   - $E = 10^9[N/m^2]$ (steel)
   - $I = \frac{1}{3}10^4[m^4]$
   - $m = 9 \times 10^4[kg]$, which is much larger than the mass of the beam, and
   - $f_w(t) = 10^4 \cos t$.

   After the design was set, the site for the platform was moved so that the depth of the water was a bit deeper, so the length has to be increased to $L = 1050[m]$. Will the platform rock more or less than originally predicted? Justify your answer.

   Interestingly, as a fluid flows through a pipe, the natural frequency of the pipe decreases. If your design assumed zero fluid flow, will the structure rock more or less than predicted when fluid is flowing through it?

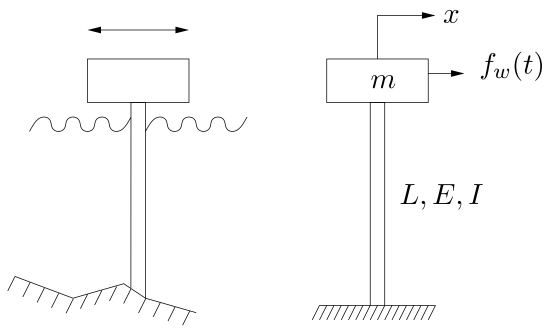

Figure 1: Offshore platform design.

[1] If it makes you feel better, you can consider the possible future scenario where carbon is sequestered underground instead of being extracted in the form of oil.

2. A piece of rotating equipment is mounted on a platform as illustrated in Figure 2. In an attempt to isolate it from the surrounding environment, it has a spring on each side.

   A rotating machine can never be perfectly balanced. Assume that experiments have determined that the horizontal force this imbalance creates on the mass is $f = m_e r\omega^2 \cos\omega t$, where ω is the angular velocity of the rotational components in the machine.

   Determine the differential equation that describes how the mass moves horizontally. Find the particular solution. Determine an expression for the force on the left wall.

   The above solutions should be in terms of k_1, k_2, etc. Now assume the following specific values: $rm_e = 100$, $m = 100$, $k_1 = k_2 = 51$, $\omega = 1$. If k_2 is increased slightly, will the magnitude of the force on the left wall increase, decrease or stay the same? If the speed of the rotation increases slightly, will the force on the left wall increase, decrease or stay the same? Justify all your answers with a mathematical analysis.

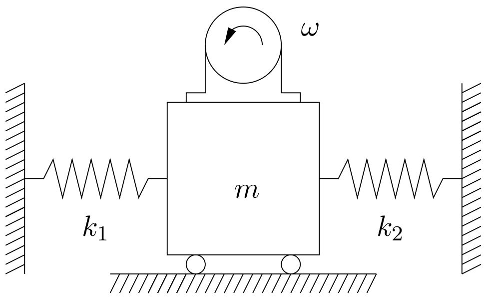

Figure 2: Motor system for Problem 2.

   Work this problem all the way out. Even if you can follow or mirror a problem in the book, put down all the work for each step.

3. Solve

$$\begin{aligned} \ddot{x} + \dot{x} + x &= 13e^{3t} + 4\cos 2t - 6\sin 2t \\ x(0) &= 1 \\ \dot{x}(0) &= 7. \end{aligned}$$

4. It is now 2030 and your lifelong dream of building satellites has been realized. In order to have the sensitive instrumentation on the satellite isolated during launch, it is mounted in the launch vehicle as illustrated in Figure 3. Consider horizontal shaking only.

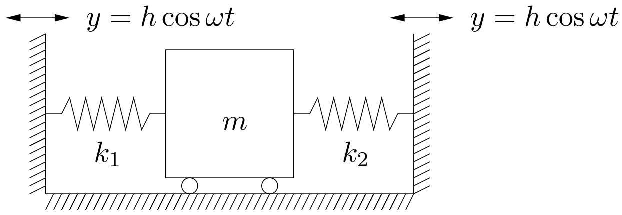

Figure 3: Satellite for Problem 4.

The satellite is the box of mass m in the Figure. During launch the whole compartment (both sides) the satellite is inside shakes back and forth by an amount $y = h\cos\omega t$, where y is measured with respect to an inertial coordinate system.

During a previous launch, a kid's science fair project that was in a similar compartment was shaken to bits. You need to decrease the *force* the satellite is subjected to during launch and are at a preliminary design review considering various options. You are the team leader and the following options are being considered:

(a) adding dampers

(b) increasing or decreasing one or both of the sprint constants

(c) redesigning the satellite to slightly increase the mass.

You need to make a decision immediately whether to pursue one of the proposed approaches or to send your team away to get more information. Sending them to get more information creates a delay and hence costs money. Thus if any of the proposed solutions is guaranteed to work, you should adopt it.

You can only send your team away if it is *impossible* to determine that any of the proposed approaches will decrease the force. The only information you have right now is what is in the problem, i.e., you don't know the specific values for the spring constants, specific values for damping constants, specific values for ω or h, etc. But maybe one of the proposed solutions will work regardless of specific values for anything.

What do you do? Justify your answer with "back of the envelope" calculations. If you refer to a book, specifically refer to equations or figures.

5. Solve

$$\begin{aligned} 2\ddot{x} + \dot{x} + 32x &= 0 \\ x(0) &= 1 \\ \dot{x}(0) &= 0. \end{aligned}$$

Sketch the solution.