University of Notre Dame
Aerospace and Mechanical Engineering

AME 301: Differential Equations, Vibrations and Controls
Exam 1

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NAME: _____

- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, your class notes and your own homework sets. You may **not** use a calculator.
- There are four problems, each is worth 25 points.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page or on the blank pages. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- Do not start or turn the page until instructed to do so.

Everyone thinks of changing the world, but no one thinks of changing himself.

— Tolstoy

1. Find the solution of

$$\ddot{x} - 3\dot{x} + 2x = 3\sin t \qquad x(0) = \frac{29}{10} \qquad \dot{x}(0) = \frac{33}{10}$$

using the method of undetermined coefficients.

2. Find the general solution of

$$\ddot{x} + 2\dot{x} + x = \frac{e^{-t}}{t}.$$

3. Consider

$$\cos(x)\ddot{x} + x^2\dot{x} + 4x = 0. \tag{1}$$

(a) Verify that the point $(x, \dot{x}) = (0, 0)$ is an equilibrium point for this equation. (5 points)

(b) Determine the best linear approximation to Equation 1 near the point $(x, \dot{x}) = (0, 0)$. (20 points)

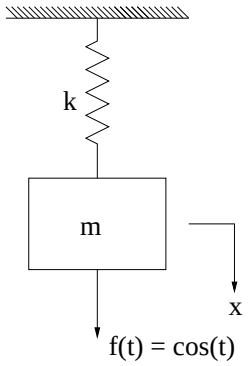

Figure 1. Spring-mass system for Problem 4.

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4. Consider the system illustrated in Figure 1.

   The equation of motion for this system is

$$m\ddot{x} + kx = \cos t.$$

   If $k = m = 1$, $x(0) = 1$ and $\dot{x}(0) = 2$, use either the method of variation of parameters or undetermined coefficients to determine the response of the system.