University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Exam 1

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NAME: _____

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course texts, any other text book, your class notes, homework solutions and your own homework sets.
- You may **not** use a calculator.
- There are four problems. Each problem is worth 25 points. Except for problem 4, *show your work* if you want to receive partial credit for any problem.
- Your grade on this exam will constitute 25% of your total grade for the course.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

To repeat what others have said, requires education; to challenge it, requires brains.
—Mary Pettibone Poole, *A Glass Eye at a Keyhole,* 1938

1. Determine the solution to

$$\begin{aligned} \dot{x}x &= \frac{1}{2} \\ x(1) &= -1. \end{aligned}$$

2. Consider

$$\ddot{x} + \dot{x} - 6x = -15\cos 3t - 3\sin 3t.$$

(a) Determine the general solution.

(b) Plot the particular solution as a function of t.

(c) If we are only concerned about the steady state solution, is is appropriate to consider only the particular solution? Justify your answer.

3. Consider the mass-spring-damper system illustrated in Figure 1.

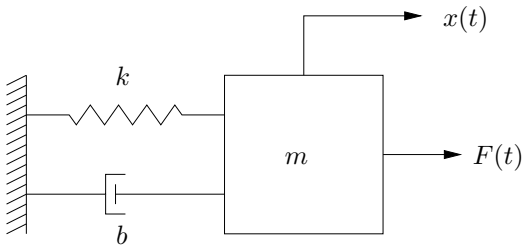

Figure 1. Mass–spring–damper system for Problem 3.

If $x(t)$ is measured from the unstretched position of the spring and

$$\begin{aligned} F(t) &= 4 + \cos 3t \\ m &= 1 \\ b &= 1 \\ k &= 4, \end{aligned}$$

determine the steady-state solution for the system.

4. Write the solution (or general solution) to each differential equation in the space provided. The first two are worth two points and each of the rest are worth three points each.

(a) $\dot{x} = 2$ $\qquad x(t) =$ _____

(b) $\dot{x} = x$ $\qquad x(t) =$ _____

(c) $\dot{x} = -x$ $\qquad x(t) =$ _____

(d) $\ddot{x} + x = 0$ $\qquad x(t) =$ _____

(e) $\ddot{x} - x = 0$ $\qquad x(t) =$ _____

(f) $\ddot{x} + \dot{x} = 0$ $\qquad x(t) =$ _____

(g) $3\ddot{x} + 4x = 0, \quad x(0) = 2, \quad \dot{x}(0) = -1$ $\qquad x(t) =$ _____

(h) $\ddot{x} + 4x = \cos 2t$ $\qquad x(t) =$ _____

(i) $\frac{d^3x}{dt^3} - x = 0$ $\qquad x(t) =$ _____