University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Exam 2

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NAME: ______________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course texts, any other text book, your class notes, homework solutions and your own homework sets.
- You may use a calculator for elementary arithmetic computations (addition, subtraction, multiplication and division) only.
- There are four problems. Each problem is worth 25 points.
- Your grade on this exam will constitute 25% of your total grade for the course.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

As with the tone-deaf, they don't know what they miss. They give a pitying chuckle at the news of scientists who have never read a major work of English literature. They dismiss them as ignorant specialists. Yet their own ignorance and their own specialization is just as startling. A good many times I have been present at gatherings of people who, by the standards of the traditional culture, are thought highly educated and who have with considerable gusto been expressing their incredulity at the illiteracy of scientists. Once or twice I have been provoked and have asked the company how many of them could describe the Second Law of Thermodynamics. The response was cold: it was also negative. Yet I was asking something which is about the scientific equivalent of: Have you read a work of Shakespeare's?

I now believe that if I had asked an even simpler question – such as, What do you mean by mass, or acceleration, which is the scientific equivalent of saying, Can you read? – not more than one in ten of the highly educated would have felt that I was speaking the same language. So the great edifice of modern physics goes up, and the majority of the cleverest people in the western world have about as much insight into it as their neolithic ancestors would have had. — C. P. Snow, The Two Cultures.

1. Compute the approximate value for the solution of

$$\begin{aligned} \dot{x} &= 3 + t - x \\ x(0) &= 1 \end{aligned}$$

at $t = 0.1$ using the fourth order Runge-Kutta method and a time step of $\Delta t = 0.1$, *i.e.*, only compute one time step. (This is problem 1 from section 8.3 of the course text).

Be sure to clearly show your work to receive any partial credit.

2. Consider

$$\begin{aligned} \dot{x} &= t^4 \\ x(0) &= 1. \end{aligned} \tag{1}$$

Figures 1 through 4 illustrate approximate numerical solutions to equation 1 using

- Euler's method,
- the second order Taylor series method,
- the second order Runge-Kutta method, and
- the fourth order Runge-Kutta method.

By only referring to the plots (*i.e.,* don't try to compute the time steps by hand), indicate below which method generated each plot and justify your answer. If it is the case that you can not determine that only one method generated the plot, then list all the methods that could have reasonably generated the solution.

(a) Figure 1: Method: ______________________________
Explanation:

(b) Figure 2: Method: ______________________________
Explanation:

(c) Figure 3: Method: ______________________________
Explanation:

(d) Figure 4: Method: ______________________________
Explanation:

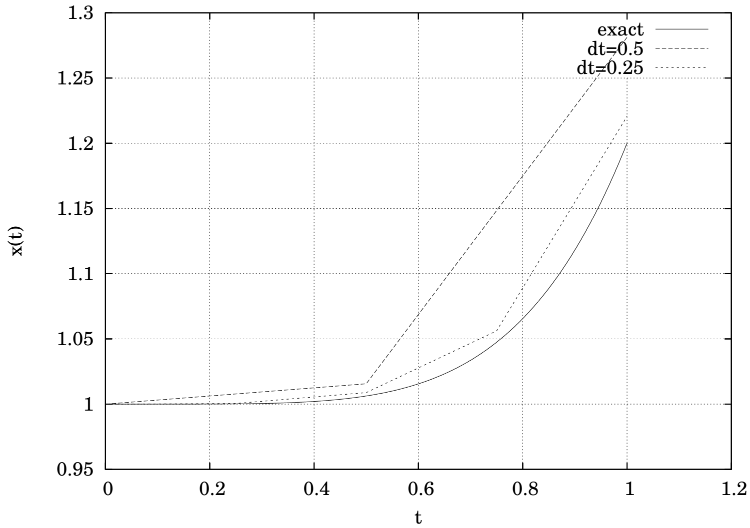

Figure 1. Exact *vs.* approximate solutions for Problem 2.

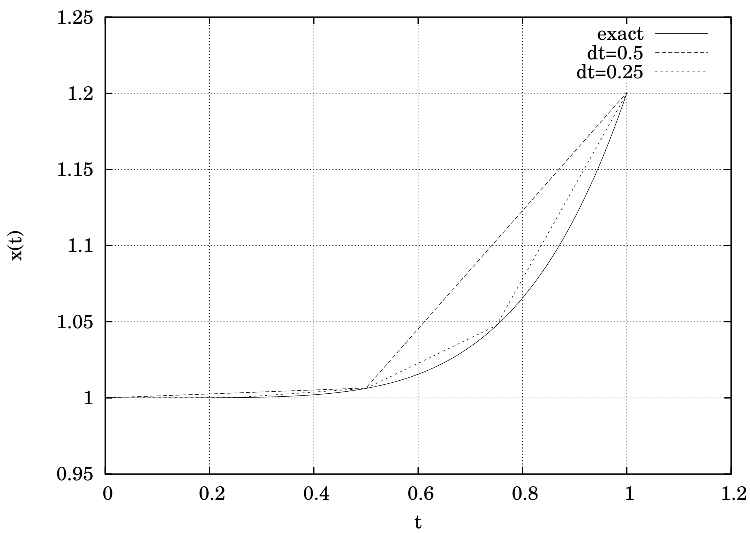

Figure 2. Exact *vs.* approximate solutions for Problem 2.

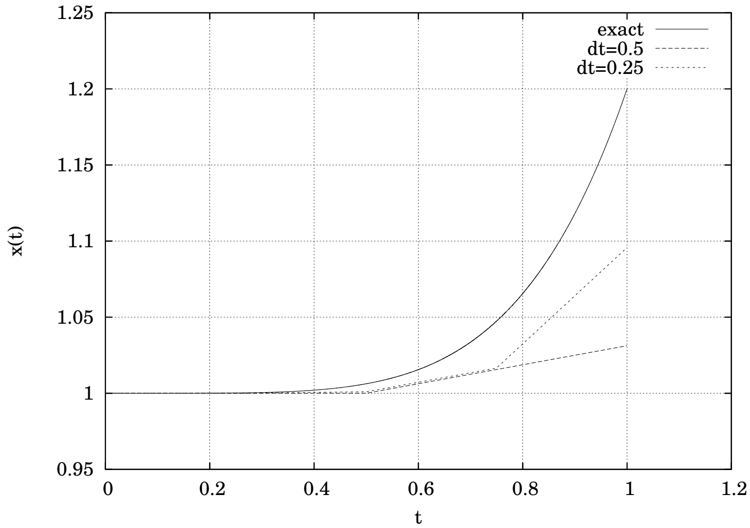

Figure 3. Exact *vs.* approximate solutions for Problem 2.

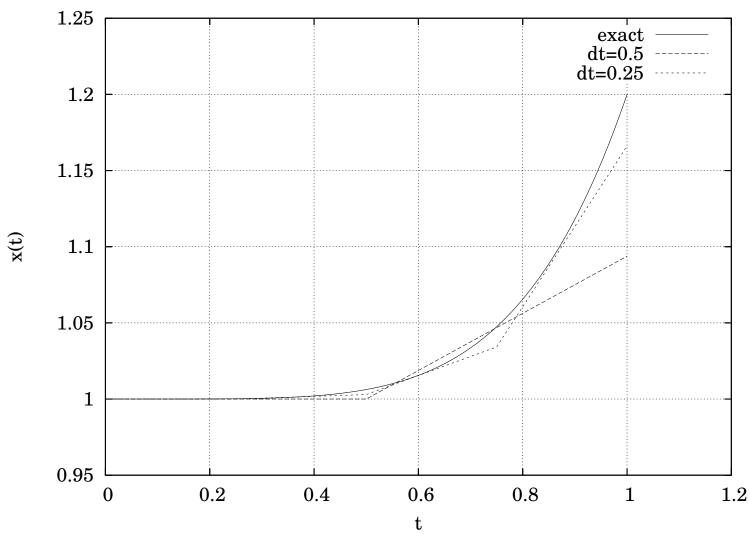

Figure 4. Exact *vs.* approximate solutions for Problem 2.

3. On the lines on the following page, write a computer program to determine an approximate numerical solution to the differential equation

$$
\begin{aligned}
\ddot{x} + t\dot{x} + (1-x)^2 x &= 0 \\
x(0) &= 1 \\
\dot{x}(0) &= 2
\end{aligned}
$$

using the second order Taylor series method.

The program must be in FORTRAN, C or C++. Syntax will not be scrutinized for miniscule errors; however, the program must generally conform to the syntax of the language used.

4. Consider a room that is being heated, governed by the following differential equation:

$$\frac{dT(t)}{dt} = \alpha(T_\infty - T(t)) + q(t)$$

where $T(t)$ is the temperature inside the room, α is a heating constant, T_∞ is the temperature surrounding the room, and $q(t)$ is the heating source. Use a PI (Proportional-Integral) controller to control the room temperature. Let $\alpha = 12$ and assume that the outside temperature is dropping exponentially according to $T_\infty(t) = 270 + 10e^{-0.02t}$ and that you desire to keep the room at a constant temperature of $T_r = 295$.

(a) (10 points) Using a proportional control constant of $K_p = 4$, determine the range of values for K_i necessary to produce an overdamped system response (i.e. No oscillations).

(b) (10 points) Let $K_p = 4$ and $K_i = 12$. Find the homogeneous solution.

(c) (10 points) Write out the assumed form of the particular solution for this problem. (DO NOT SOLVE)